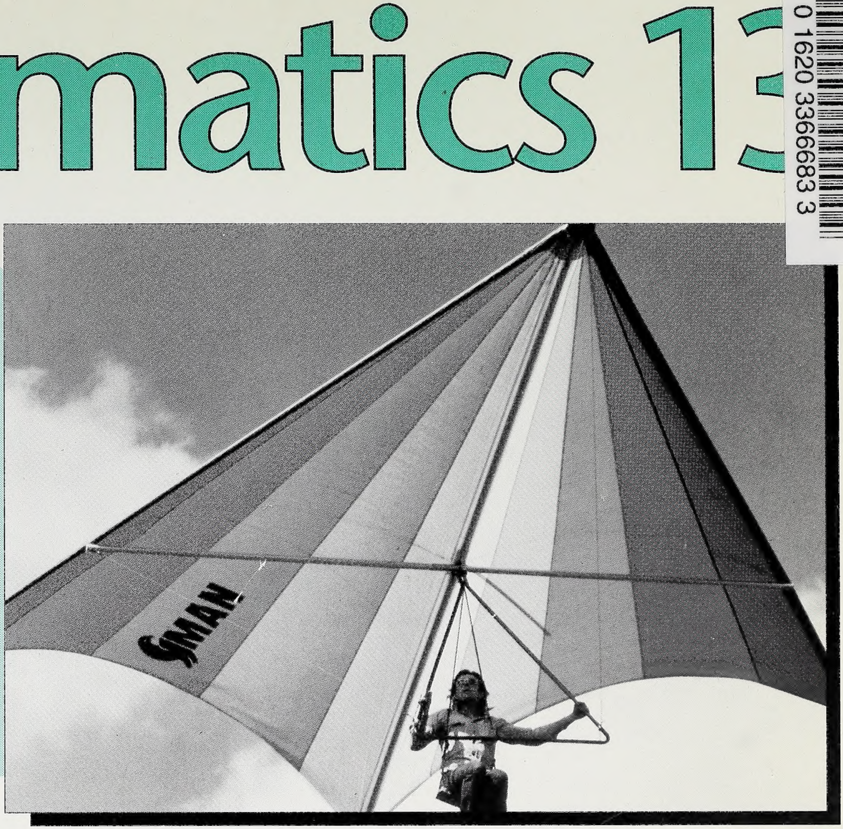
Mathematics 13



UNIT 3
Geometry

Distance Learning
Alberta
EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 13 Student Module Unit 3 Geometry Alberta Distance Learning Centre ISBN No. 0-7741-0588-7

Cover photo: PHOTO SEARCH LTD.

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

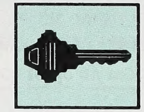
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

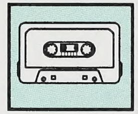
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

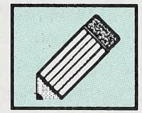
What You Already Know
- reviewing what you already know



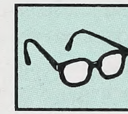
Key Idea
- flagging important ideas



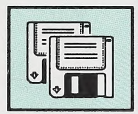
Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts



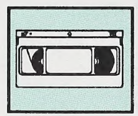
Another View
- exploring different perspectives



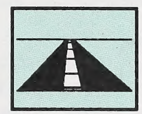
Computer Software
- learning by using computer software

Introduction
- introducing the unit

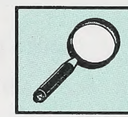
Solutions
- correcting the activities



Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit

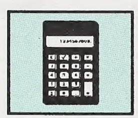


Extra Help
- providing additional study

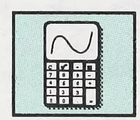
Print Pathway
- choosing a print alternative

Exploring the Topic
- actively learning new concepts

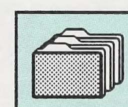
Extensions
- going on with the topic



Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 13

Course Overview

Mathematics 13 contains 6 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Polynomials and Factoring	28%
Unit 3 Geometry	18%
Unit 4 Coordinate Geometry and Graphing	18%
Unit 5 Relations	12%
Unit 6 Statistics	14%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Geometry

This unit covers topics dealing with geometry. Each topic contains explanations, examples, and activities to assist you in understanding geometry. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 3 Geometry

Contents at a Glance

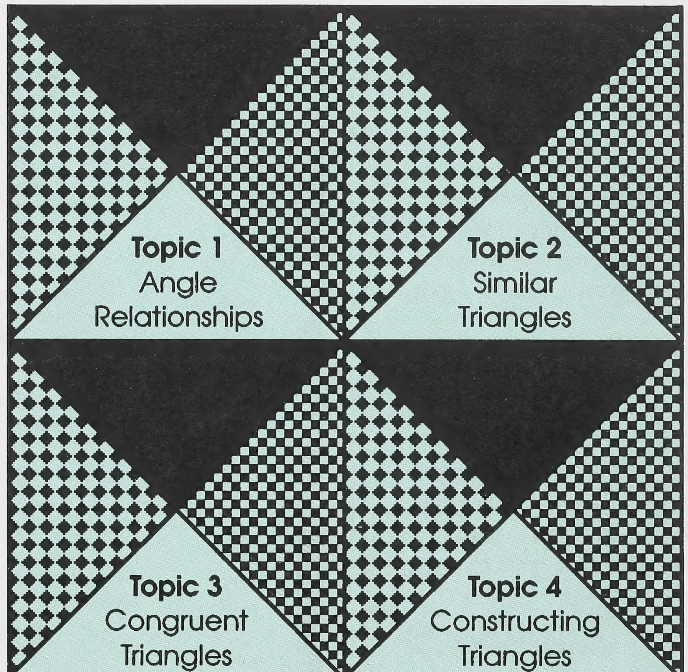
Geometry

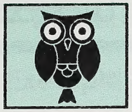
What do geology, geography, and geometry have in common? The three words start with **geo**. Geology is the study of the earth's crust and rocks. Geography is the study of the earth's surface. How is geometry related to a study of the earth?

Geometry was developed by the Egyptians to help locate plots of land after the annual flooding of the Nile. In fact, the word **geometry** means **earth measure**.

Today geometry involves the study of lines, angles, triangles, and rectangles among other things. This is done not just for practical reasons, but also for the sake of just studying geometry. In this unit you will be able to appreciate geometry and see it in actual application.

Unit 3 Geometry





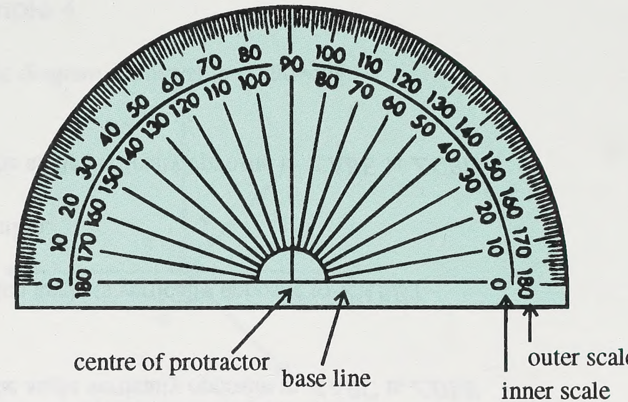
What You Already Know

Recall the following.

- Vertically opposite angles are also called vertical angles.
- When naming angles using three letters, the letter of the vertex is placed in the middle.
- In any triangle, the sum of the measures of the three angles is always 180°.
- A straight angle is an angle whose rays form a straight line.

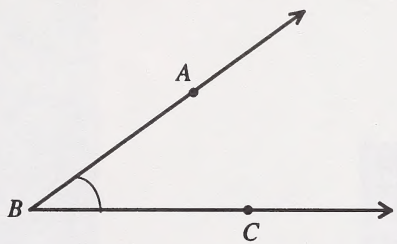
Each of the following facts is illustrated with an example. Review them carefully.

- The protractor can be used to measure an angle. An angle is measured in degrees. A protractor can measure up to 180°.



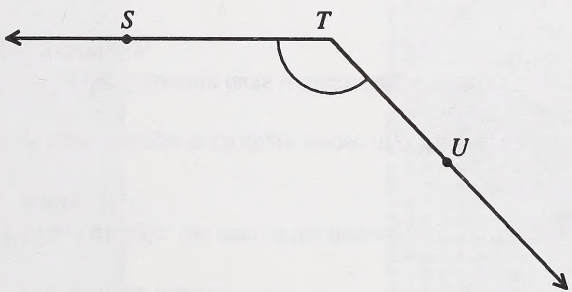
Example 1

Give the measure of the following angles.



Solution:

$\angle ABC = 36°$



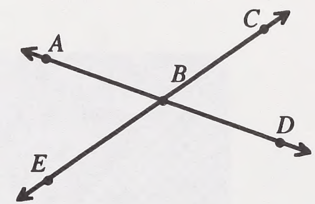
Solution:

$\angle STU = 134°$

- Vertically opposite angles are formed by two intersecting lines.

Example 2

Refer to the diagram for the following.



- Which angle is vertically opposite to $\angle ABC$?

Solution:

The angle vertically opposite to $\angle ABC$ is $\angle DBE$.

- Which angle is vertically opposite to $\angle ABE$?

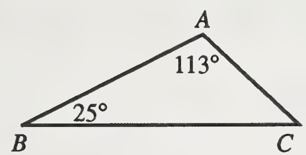
Solution:

The angle vertically opposite to $\angle ABE$ is $\angle CBD$.

- The sum of all the angles in a triangle is 180°.

Example 3

In ΔABC, $\angle A = 113°$ and $\angle B = 25°$. What is $\angle C$?



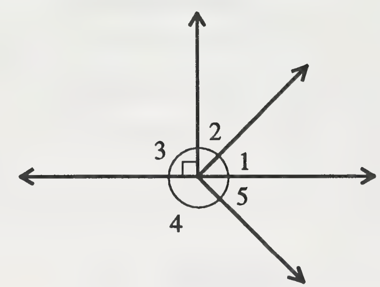
Solution:

$$\begin{aligned} \angle A + \angle B + \angle C &= 180° \\ \angle C &= 180° - (113° + 25°) \\ &= 180° - 138° \\ &= 42° \end{aligned}$$

- Two angles are supplementary if their angle measures add to 180°. Two angles are complementary if their angle measures add to 90°.

Example 4

Use the diagram to answer the following.



- Give two angles that are complementary.

  Solution:

  Since angle 3 measures 90°, the angle measures of angle 1 and angle 2 total 90°. Angle 1 and angle 2 are complementary angles.

- Give two angles that are supplementary.

  Solution:

  Angle 4 and angle 5 make a straight angle so the measures of angle 4 and angle 5 total 180°. Angle 4 and angle 5 are supplementary angles.

- A line segment can be constructed congruent to a given line segment.

Example 5

Construct a line segment congruent to $\overline{AB}$ which is given. Label the new line segment as $\overline{CD}$.

Solution:

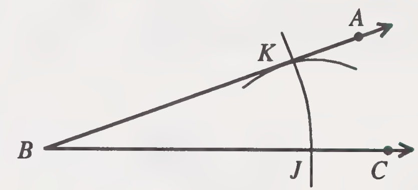
Draw a line segment at least as long as $\overline{AB}$ and label one endpoint C.

Using a compass, mark off the distance from A to B with an arc. Without changing the compass distance, place the compass point at C and make the same arc intersecting the second line segment at D.

- An angle can be constructed congruent to a given angle.

Example 6

Construct $\angle DEF$ congruent to $\angle ABC$ which is given.

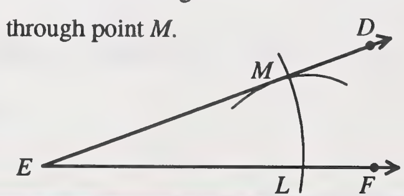


Solution:

Step 1: Draw a ray at least as long as $\overrightarrow{BC}$ and label it $\overrightarrow{EF}$.

Step 2: Set the compass point at point B and make an arc intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at K and J. Without changing the compass distance, put the compass point at E and make the same arc intersecting $\overrightarrow{EF}$ at L.

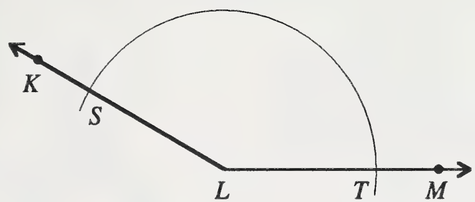
Step 3: Set the compass point at J and make an arc so that it will intersect $\overrightarrow{BA}$ at the point K. Without changing the compass distance, put the compass point at L and make an arc intersecting the other arc at M. Draw $\overrightarrow{ED}$ passing through point M.



- The bisector of an angle can be constructed.

Example 7

Bisect $\angle KLM$ which is shown. Let the bisector be $\overrightarrow{LB}$.



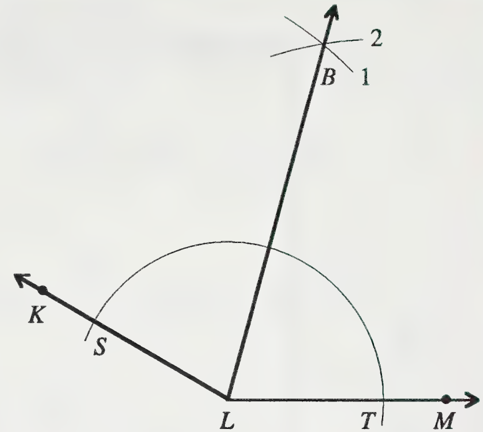
Solution:

Step 1: With the compass at point L, make an arc that intersects $\overrightarrow{LK}$ and $\overrightarrow{LM}$ at S and T.

Step 2: Extend your compass a bit, and from S and T, make arcs 1 and 2 intersecting at B.

Step 3: Draw a ray from L through point B.

Thus, $\overrightarrow{LB}$ bisects $\angle KLM$.



Use a protractor to check if $\angle KLB$ is congruent to $\angle MLB$. If these angles are congruent, $\overrightarrow{LB}$ is the bisector of $\angle KLM$.

- The missing element in a proportion can be calculated.

Example 8

- Solve for x in $\frac{x}{12} = \frac{10}{24}$.

  Solution:

  $$\frac{x}{12} = \frac{10}{24}$$

  $\div 2$

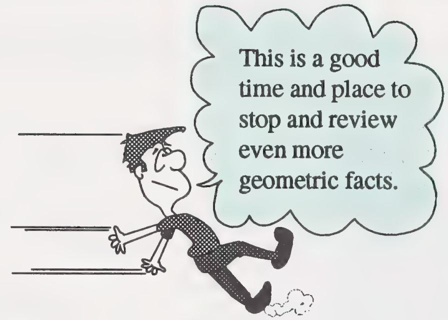
  $\div 2$

  $$\begin{aligned} x &= 10 \div 2 \\ &= 5 \end{aligned}$$

- Solve for x in $\frac{1.5}{4.5} = \frac{x}{3.0}$.

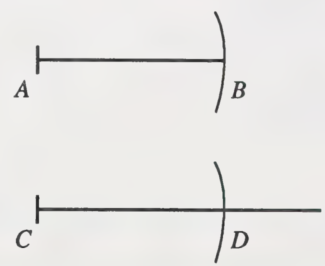
  Solution:

  $$\begin{aligned} 1.5 \times 3.0 &= 4.5x \\ 4.5x &= 4.5 \\ x &= \frac{4.5}{4.5} \\ &= 1.0 \end{aligned}$$



Do you remember the following?

- Two line segments are congruent if they have the same length.
- To construct in geometry, draw using only a compass and a straightedge.
- The arc on $\overline{CD}$ indicates that the distance measured on line segment CD is the same as the distance measured on $\overline{AB}$.



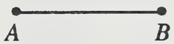
To solve for x in $\frac{x}{12} = \frac{10}{24}$, cross products are found to get an equation. The equation is then solved for x.

$$\frac{x}{12} = \frac{10}{24}$$

$$\frac{x}{12} = \frac{10}{24}$$

$$\begin{aligned} 24(x) &= (12)(10) \\ 24x &= 120 \\ \frac{24x}{24} &= \frac{120}{24} \\ x &= 5 \end{aligned}$$

- Line segments are parts of a line with two endpoints.

  For example, 

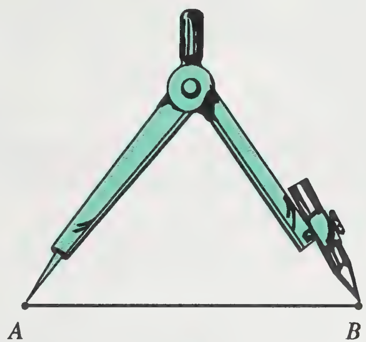
is named $\overline{AB}$.

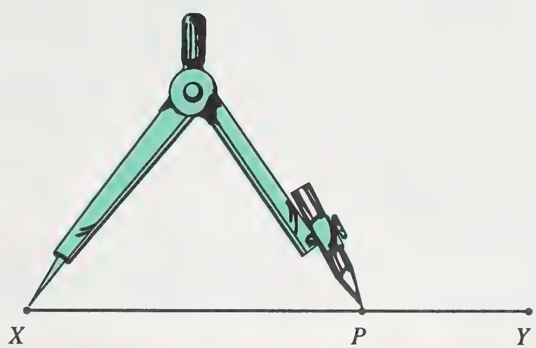
- Rays are parts of a line with one endpoint.

  For example, 

is named $\overrightarrow{AB}$.

- In



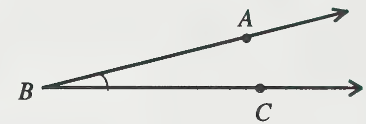


$\overline{AB} \cong \overline{XP}$ and the symbol $\cong$ means congruent.

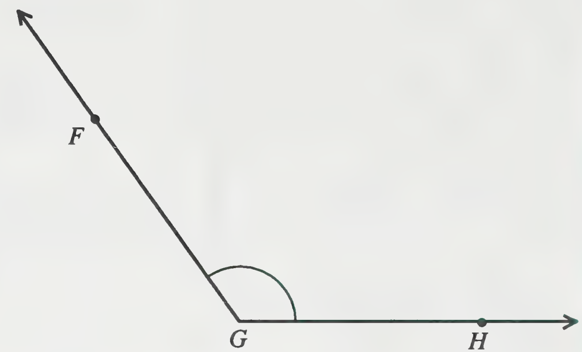
Review

1. Measure the following angles.

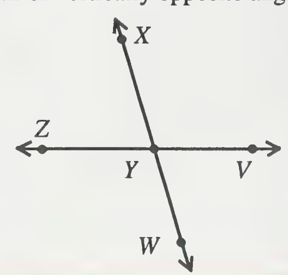
   a.



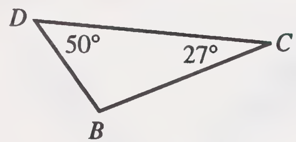
   b.



2. Name a pair of vertically opposite angles in the diagram.



3. In the following triangle, the measures of two of the angles are shown. What is the measure of $\angle B$?



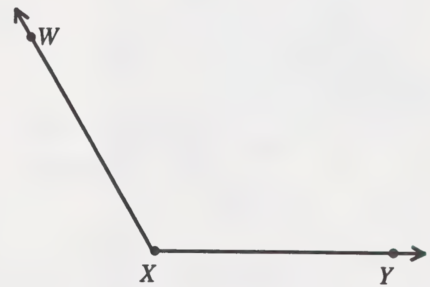
4. Suppose that $\angle A$ and $\angle B$ are complementary, $\angle B$ and $\angle E$ are supplementary, and $\angle B = 85°$.

   a. What is the measure of $\angle A$?

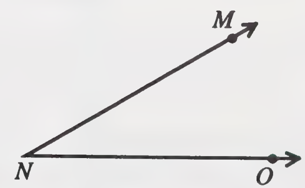
   b. What is the measure of $\angle E$?

5. Construct a line segment congruent to $\overline{MN}$ which is shown. Label your line segment $\overline{PQ}$.

6. Construct an angle congruent to $\angle WXY$ which is shown. Label your angle $\angle TUV$.



7. Bisect $\angle MNO$.



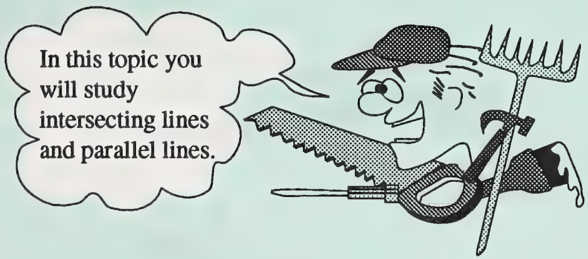
Now go to the **Review** solutions in **Appendix A**.

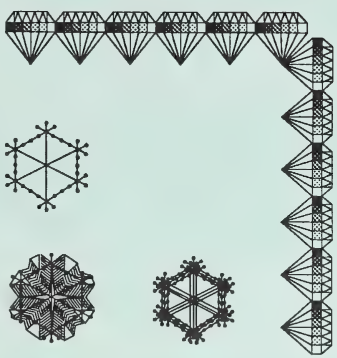
If you had trouble with these questions, go to **Mathematics 9, Module 5**.

Remember that when you are asked to construct an angle, you are to use a compass and a straightedge only.

Topic 1 Angle Relationships

Introduction





You also will discover how the angles formed by these lines relate to each other.

What Lies Ahead

Throughout this topic you will learn to

1. recognize the relationship between pairs of vertically opposite angles and how to use this relationship to calculate the size of angles

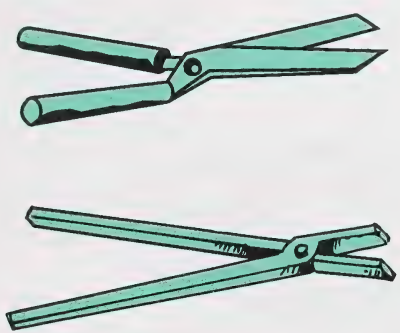
2. recognize the relationship among the angles made by cutting parallel lines with a transversal and how this relationship can be applied

Now that you know what to expect, turn the page to begin your study of angle relationships.

Exploring Topic 1

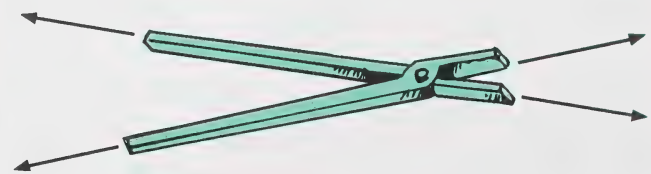
Activity 1

Recognize the relationship between pairs of vertically opposite angles and how to use this relationship to calculate the size of angles.



Look at the tools pictured. How does the angle made by the handles compare to the angle made by the front part of these tools? It seems that for the shears the angle of the handles is always larger than the angle of the blades. The tool is probably designed in this way so that you do not hurt your knuckles during the clipping motion. Notice that the blade and the handle are not on the same line.

What about the tongs? The handles are much longer than the jaws, but you know that does not affect the angle size. Use the rays drawn on the diagram to compare the angle sizes with your protractor.



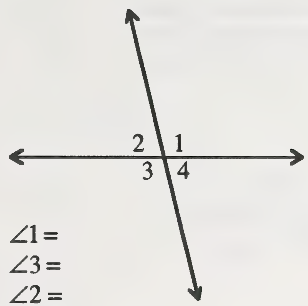
You should find that the angle made by the handles is about 20° and that the angle made by the jaws is also 20°. For the tongs, the handle angle is the same size as the jaw angle. Here the jaw and the handle do fall on the same line.

For the shears and the tongs you compared angles. In a sense the angles are opposite with respect to the pivoting pin. When two straight lines intersect, **vertically opposite** angles are formed. In the following activity you will discover the angle measure relationship for vertically opposite angles.

Do the following questions.

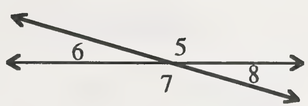
1. For the intersecting lines shown, measure the angles formed. You may want to extend the lines to help you measure the angles with the protractor. List the angle measurements to show how they compare for vertically opposite angles.

a.



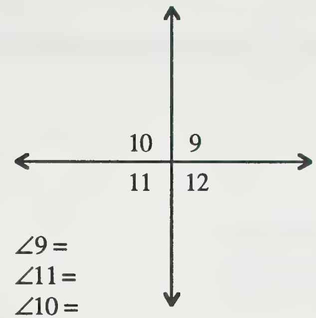
$\angle 1 =$
$\angle 3 =$
$\angle 2 =$
$\angle 4 =$

b.



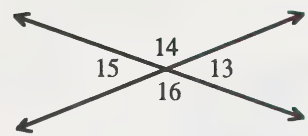
$\angle 5 =$
$\angle 7 =$
$\angle 6 =$
$\angle 8 =$

c.



$\angle 9 =$
$\angle 11 =$
$\angle 10 =$
$\angle 12 =$

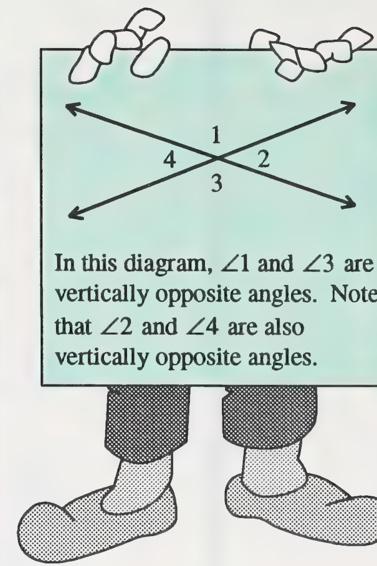
d.



$\angle 13 =$
$\angle 15 =$
$\angle 14 =$
$\angle 16 =$

2. From the measurements you made in question 1, make a generalization about vertically opposite angles. Allow for measurement inaccuracies.

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.



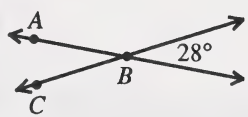
From the practice questions you found that vertically opposite angles have the same measure. This can be expressed as follows.

Vertically opposite angles are equal in measure.

This relationship can be used along with other geometric facts you already know to find the size of certain angles.

Example 1

Find the measure of $\angle ABC$.

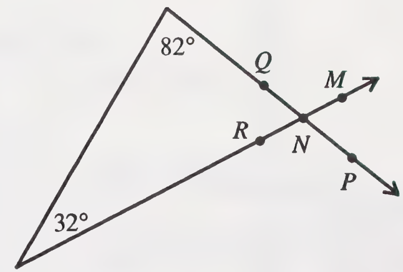


Solution:

Angle ABC is vertically opposite to the angle measuring 28°. Since vertically opposite angles are equal in measure, $\angle ABC = 28°$.

Example 2

What is the measure of $\angle MNP$?



Solution:

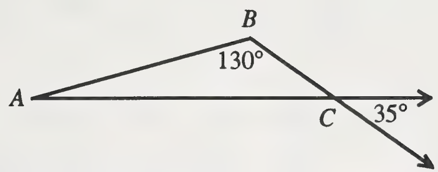
The sum of the angles of any triangle is 180°.

$$\begin{aligned}\angle QNR &= 180° - (32° + 82°)\\ &= 180° - 114°\\ &= 66°\end{aligned}$$

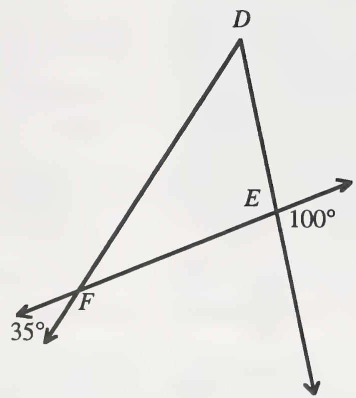
Since $\angle MNP$ and $\angle QNR$ are vertically opposite angles, they are equal in measure. Therefore, $\angle MNP = 66°$.

You should now be able to apply the vertically opposite angle relationship in the questions that follow.

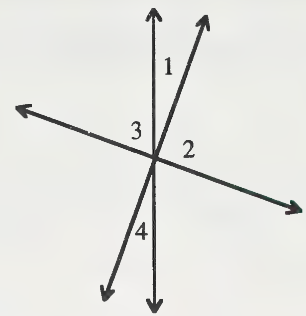
3. Determine the measure of $\angle A$. Use the angle measures shown.



4. Using the angle measures shown, determine the measure of $\angle D$.



5. Refer to the diagram where $\angle 2 = 90°$ and $\angle 3 = 70°$. Determine the measure of $\angle 4$.



For solutions to **Activity 1**, turn to **Appendix A**, **Topic 1**.

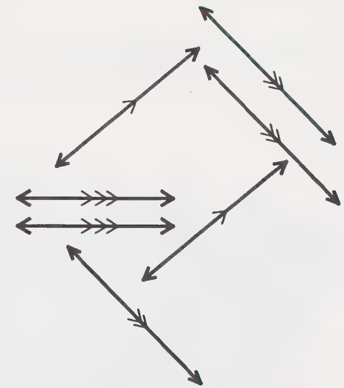
In some of these questions you will also use your knowledge of complementary and supplementary angles.

Activity 2

Recognize the relationship among the angles made by cutting parallel lines with a transversal and how this relationship can be applied.

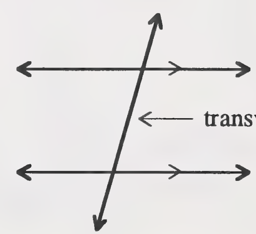
Parallel lines are lines that stay the same distance apart. In the photo the tracks seem to converge or meet in the distance, but you know they must stay the same distance apart throughout. The tracks must be always separated by the same distance as the wheels are separated on each train axle. Therefore, you know that the tracks must be parallel.

The next diagram shows examples of parallel lines. **Parallel lines never meet.** Notice how the > symbol is used to show which lines are parallel with each other. In two cases there are two parallel lines, and in one case there are three parallel lines.



Parallel lines do not meet so they do not make angles like intersecting lines do. However, when another line cuts parallel lines, several angles are formed. These angles have some interesting relationships.

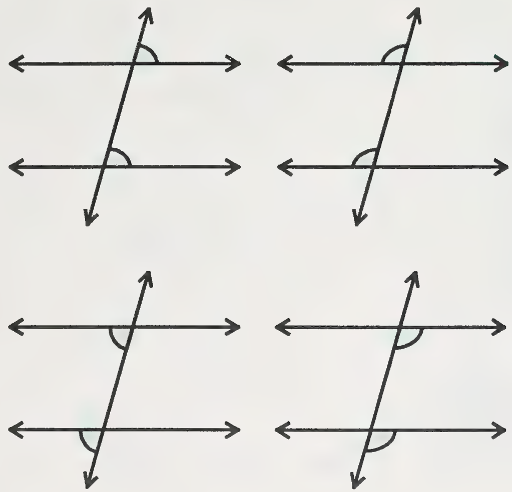
Before you explore these angle relationships, it would be helpful to introduce some terminology.



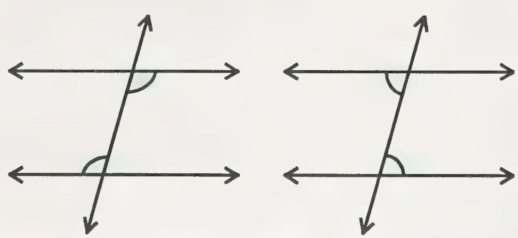
The **transversal** is **the line intersecting two or more parallel lines**. In the picture of the railway tracks, the railway ties can be thought of as transversals for the parallel tracks.

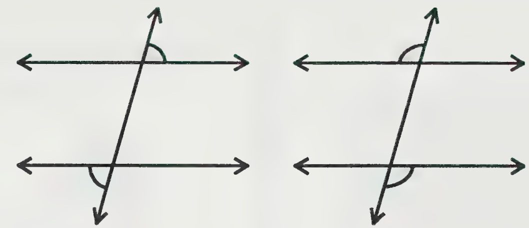
Corresponding, Alternate, Interior, and Exterior Angles

In each of the following, the shading shows pairs of **corresponding angles**. In each case the two corresponding angles are equal in measure.

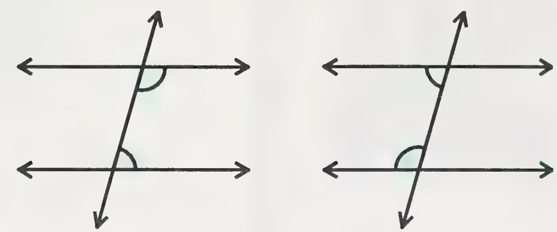


In the following, shading is used to point out the pairs of **alternate angles**. In each case the two alternate angles are equal in measure.

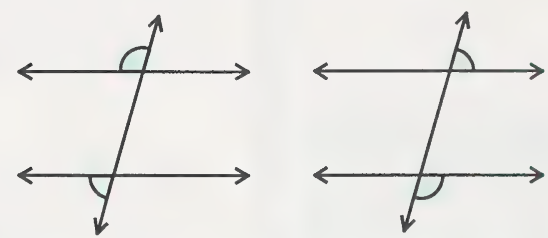




In the following, shading is used to show the **interior angles**. Interior angles on the same side of the transversal are supplementary.

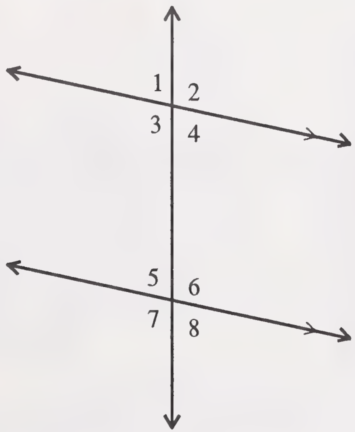


In the following, the **exterior angles** are shaded. Exterior angles on the same side of the transversal are supplementary.



You can determine these angle relationships in the questions that follow. With the new terminology, you should be able to apply your findings.

1. Give the measure for each of the angles labelled. Use your protractor.



$\angle 1 =$ $\qquad$ $\angle 2 =$

$\angle 3 =$ $\qquad$ $\angle 4 =$

$\angle 5 =$ $\qquad$ $\angle 6 =$

$\angle 7 =$ $\qquad$ $\angle 8 =$

2. What is the relationship between two corresponding angles? Give evidence to support your answer.

3. Tell how alternate interior angles compare. Support your answer.

4. Now look at the interior angles. Can you find a relationship for interior angles on the same side of the transversal? If there is a relationship, tell what it is and support your answer.

5. You have found certain angle relationships in the previous questions. Make your own pair of parallel lines with a transversal. Using the angles formed, verify the relationships you have found.

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

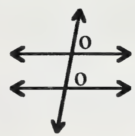
You can make parallel lines by outlining the two long edges of a ruler or by using the edges of an audiocassette case or some other such object.

Interior angles on the same side of the transversal are also called same-side interior angles. The same applies to exterior angles on the same side of the transversal. They are called same-side exterior angles.

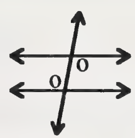
One pair of alternate interior angles is $\angle 3$ and $\angle 6$. Another pair of alternate interior angles is $\angle 4$ and $\angle 5$.

The angle relationships for parallel lines and a transversal can be listed as follows:

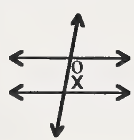
- Corresponding angles have the same measure.



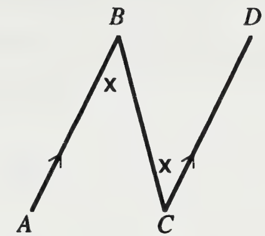
- Alternate interior angles have the same measure.



- Same-side interior angles are supplementary.

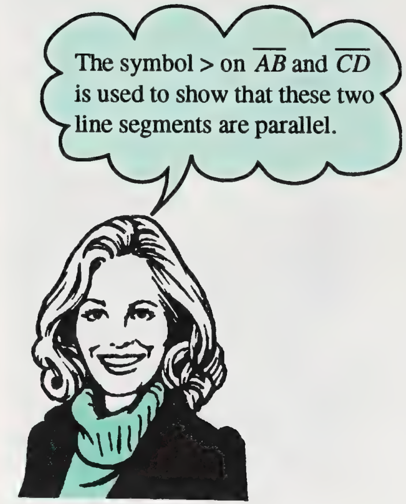


The transversal and the related angles may not be easy for you to see in some figures. Look at the following figure as a typical example.



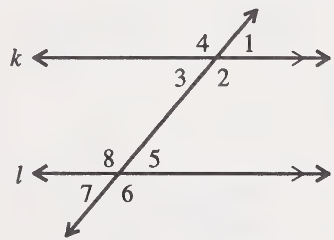
Note that $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{BC}$ is the transversal, and $\angle ABC$ and $\angle BCD$ are alternate interior angles having the same measure.

The following examples show how angle relationships can be used to find the size of other angles.



Example 3

In the following figure, line $k \parallel$ line l. In the diagram, $\angle 8 = 130°$ and $\angle 5 = 50°$. Without using a protractor, find the measures of the remaining angles. Give reasons for your answers.



Solution:

$\angle 8 = 130°$ (given)

$\angle 6 = 130°$ (Since they are vertically opposite angles, $\angle 8 = \angle 6$.)

$\angle 5 = 50°$ (given)

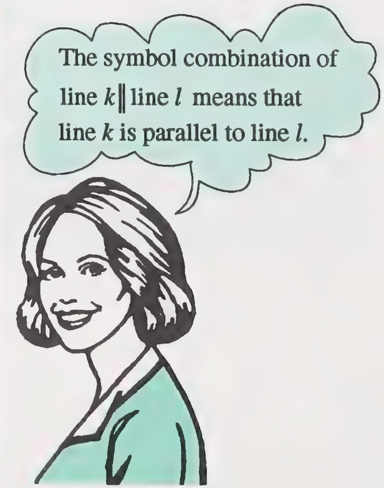
$\angle 7 = 50°$ (Since they are vertically opposite angles, $\angle 5 = \angle 7$.)

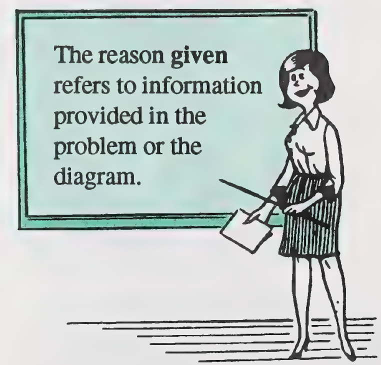
$\angle 1 = 50°$ (Since they are corresponding angles, $\angle 1 = \angle 5$.)

$\angle 2 = 130°$ (Since they are corresponding angles, $\angle 2 = \angle 6$.)

$\angle 3 = 50°$ (Since they are corresponding angles, $\angle 3 = \angle 7$.)

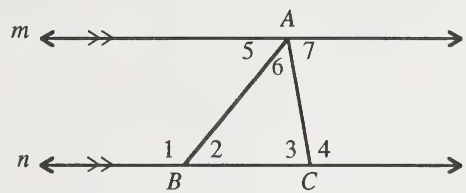
$\angle 4 = 130°$ (Since they are corresponding angles, $\angle 4 = \angle 8$.)





Example 4

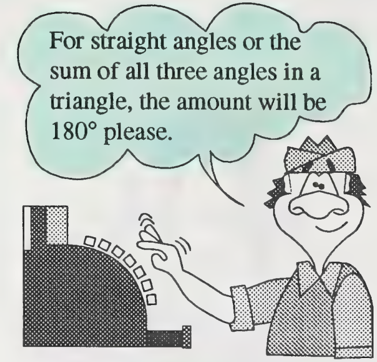
In the following diagram, $\angle 7 = 80°$ and $\angle 1 = 130°$, and $\overline{AB}$ and $\overline{AC}$ are transversals for lines m and n. Without using a protractor, find the measures of the remaining angles. Give reasons for your answers.



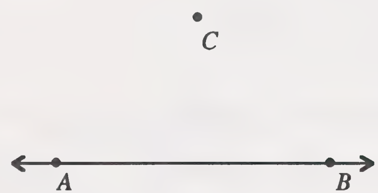
Solution:

$\angle 1 = 130°$	(given)
$\angle 2 = 50°$	(Because they form a straight angle, $\angle 1$ and $\angle 2$ are supplementary.)
$\angle 5 = 50°$	(Because they are alternate angles, $\angle 2$ and $\angle 5$ are equal.)
$\angle 7 = 80°$	(given)
$\angle 4 = 100°$	(Because they are same-side interior angles, $\angle 4$ and $\angle 7$ are supplementary.)
$\angle 3 = 80°$	(Because they form a straight angle, $\angle 3$ and $\angle 4$ are supplementary.)
$\angle 6 = 50°$	(The sum of angles in a triangle is 180°.)

$$\begin{aligned} \angle 2 + \angle 3 + \angle 6 &= 180° \\ 50° + 80° + \angle 6 &= 180° \\ 130° + \angle 6 &= 180° \\ \angle 6 &= 180° - 130° \\ \angle 6 &= 50° \end{aligned}$$

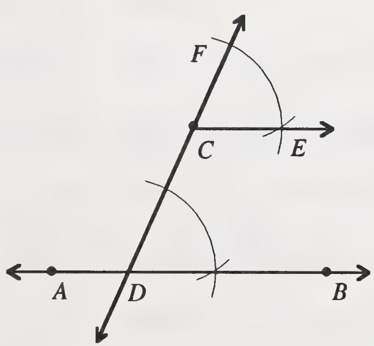


Can you use your knowledge of angle relationships to construct parallel lines? Suppose you want to construct a line through point C which is parallel to $\overleftrightarrow{AB}$.

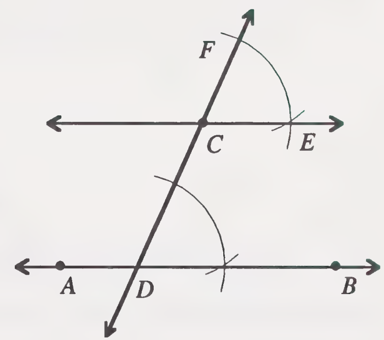


Step 1: Draw $\overleftrightarrow{DF}$ to intersect $\overleftrightarrow{AB}$ and pass through C.

Step 2: Then $\angle FCE$ is constructed congruent to $\angle CDB$.



Step 3: Extend $\overrightarrow{CE}$ to form $\overleftrightarrow{CE}$.



Why do you think $\overleftrightarrow{CE}$ is parallel to $\overleftrightarrow{AB}$? Note that $\overleftrightarrow{FD}$ is the transversal of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CE}$. Corresponding angles were constructed to have the same measure. You know that parallel lines intersected by a transversal form congruent corresponding angles.

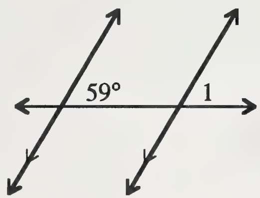
The questions that follow give you a chance to apply the angle relationships you have learned.

Recall that in construction you can use only a straightedge and a compass.

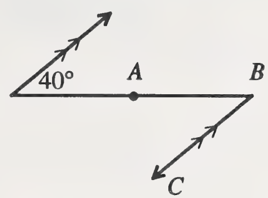
Review the construction of congruent angles in Example 6 of the **What You Already Know** section.

Do at least the odd-numbered questions. Do these questions without measuring, and give reasons for your answers.

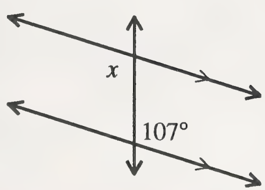
6. What is the measure of $\angle 1$?



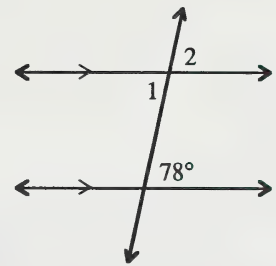
7. What is the measure of $\angle ABC$?



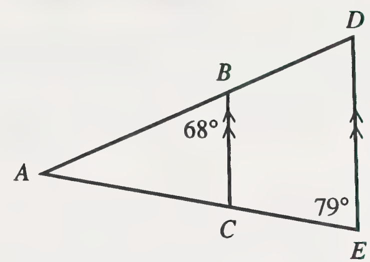
8. Find the angle marked by x.



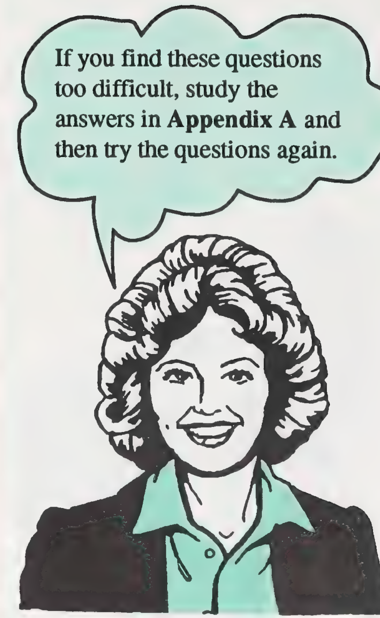
9. a. What is the measure of $\angle 2$?
   b. What is the measure of $\angle 1$?



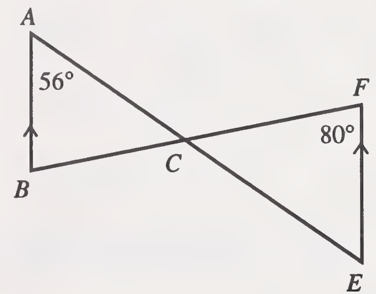
10. What is the measure of $\angle BAC$?



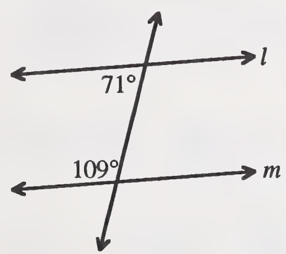
11. What is the measure of $\angle CBD$?



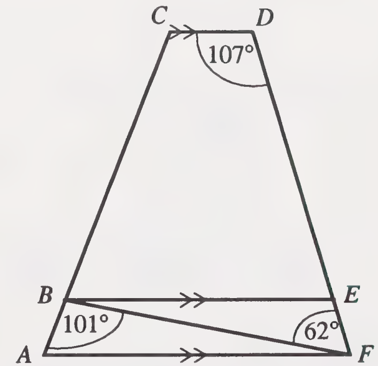
12. What are the measures of $\angle DCE$ and $\angle ACB$?



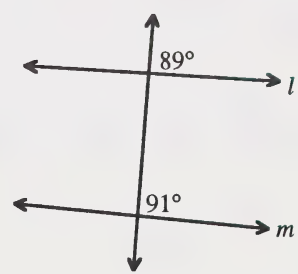
13. Are lines l and m parallel?

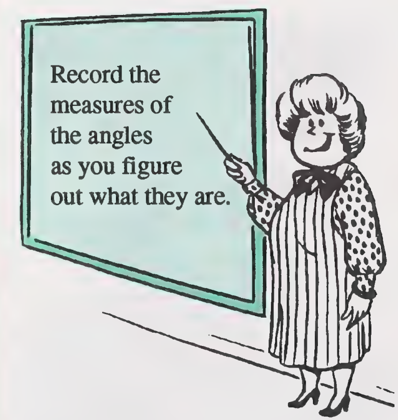


14. Suppose that $\angle CDE = 107°$, $\angle EFB = 62°$, and $\angle ABF = 101°$. What is the measure of $\angle BCD$?

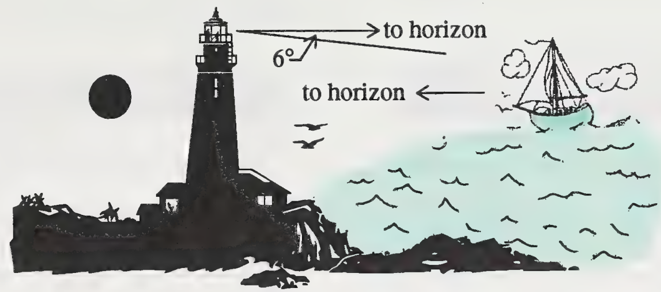


15. Are l and m parallel?





16. The person in the lighthouse sees a boat in the distance. This person notes that it is 6° below the horizon. To the sailor, how many degrees above the horizon would the lighthousekeeper appear to be?



For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

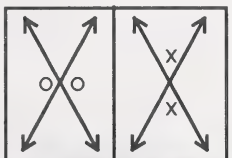
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

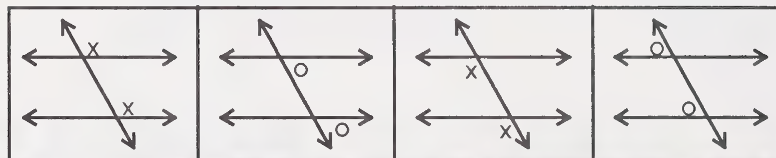
You may decide to do both.

Extra Help

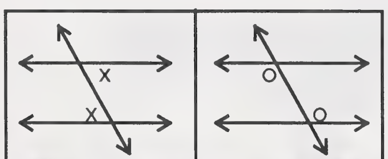
In this topic you studied angle relationships for intersecting lines and parallel lines. You may have found it difficult to keep the relationships and the corresponding terms straight. It may help to see the relationships and the names of the angles such that comparisons may be made.



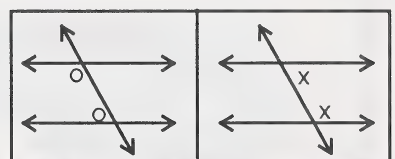
Vertically opposite angles



Corresponding angles



Alternate interior angles



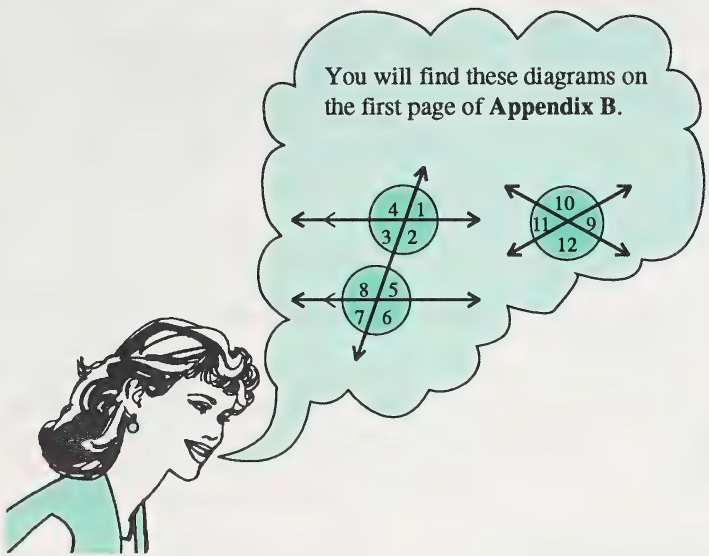
Same-side interior angles

The following activity should help to show you angle relationships.

Questions 1 to 4 refer to the angles drawn on the sheet of manipulatives for Topic 1 found in **Appendix B**.

1. Remove the sheet entitled **Appendix B Manipulatives, Angles for Topic 1, Extra Help**. Cut along the circle surrounding angles 5, 6, 7, and 8. Then cut along the lines within the circle. Place sector 5 on sector 1, sector 6 on sector 2, sector 7 on sector 3, and sector 8 on sector 4. What does this tell you about the angle relationship for corresponding angles?

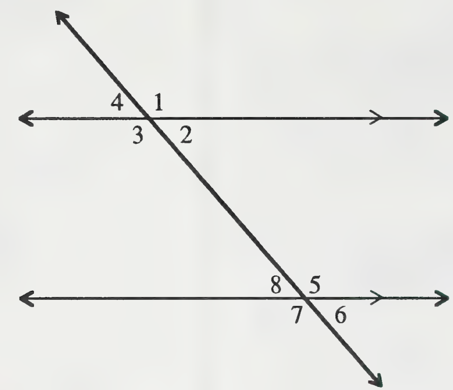
2. Cut out sectors 2 and 3. Place sectors 2 and 5 beside each other so that they touch along the sides and at the point. Place sectors 3 and 8 together in the same way. What does this tell you about same-side interior angles?



3. Place sectors 2 and 8 on top of each other. Do the same for sectors 3 and 5. What does this tell you about alternate interior angles?

4. Cut along the circle surrounding sectors 9, 10, 11, and 12. Place sectors 9 and 11 on top of each other. Do the same for sectors 10 and 12. What does this tell you about vertically opposite angles?

5. Suppose $\angle 1 = 131°$. Give the measures of the other angles and state reasons for your answers.

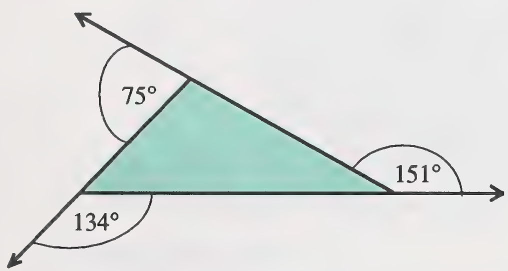


For solutions to **Extra Help**, turn to **Appendix A, Topic 1**.

Extensions

So far you have studied a few angle relationships. You already know that the three angles of a triangle add to 180°. There are still many other angle relationships. You can study a few in this section.

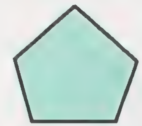
Look at the angles made by **extending** the sides of a triangle.



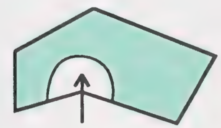
Notice that the measures of these angles add to 360°.

Exterior angles can be made for all polygons. These are angles made by a side and the extension of the adjacent side. You can see that for any triangle, the sum of the exterior angles is 360°. Does this hold true for other polygons?

Convex polygons do not have interior angles greater than 180°. For example, in the following convex polygon all interior angles are less than 180° in measure.

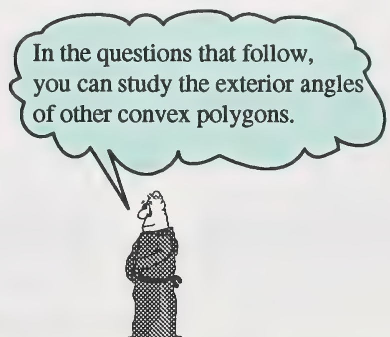


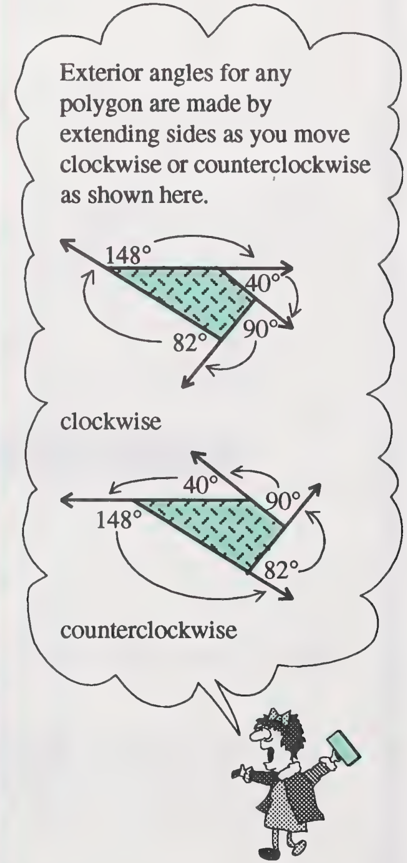
In a **nonconvex** or **concave** polygon there is an interior angle which is **greater than** 180° in measure.



This interior angle is greater than 180°.

Keep these differences in mind.



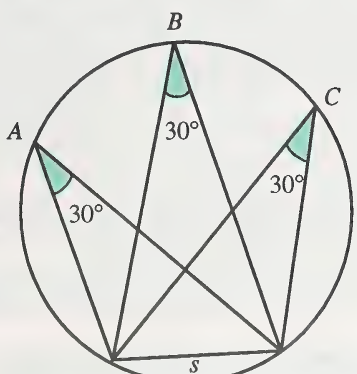


1. Draw a convex quadrilateral. Extend its sides as was done for the triangle. Measure the exterior angles and give the sum of the measurements.

2. Make a prediction about the sum of exterior angles for any convex polygon. To test your prediction, draw and use a convex five-sided polygon.

For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

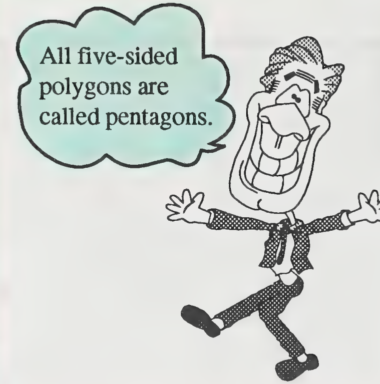
You can also explore angles and circles. In the following diagram, angles *A*, *B*, and *C* have their vertices on the same circle. For each of these angles the sides connect with the endpoints of the same chord *s*. The angles are equal in measure.



In the next question you can learn about similar angles when the sides connect with the endpoints of a special chord called a **diameter**.

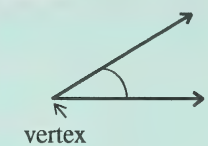
3. Use your compass to make a circle at least 10 cm across. Put in the diameter. Draw an angle having a vertex on the circle and sides that connect with the endpoints of the diameter. Measure the angle. Make several other angles in the same way. What conclusion can you make about an angle whose vertex is on a circle and whose sides connect with the diameter endpoints?

For solutions to **Extensions**, turn to **Appendix A, Topic 1**.



The side marked *s* is called a **chord** because the segment has both endpoints on the circle.

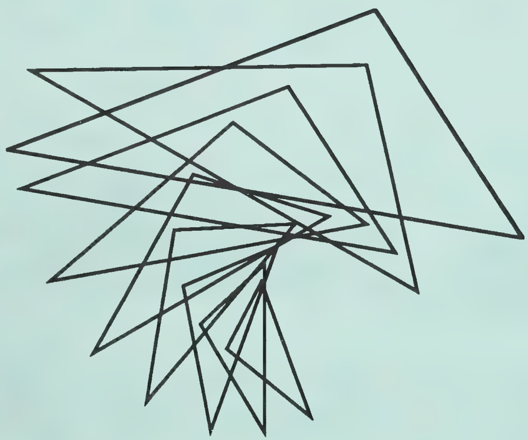
This diagram shows what a vertex is.



Remember that the plural for vertex is **vertices**.

Topic 2 Similar Triangles

Introduction



Triangles that have the same shape are called **similar triangles**.

Knowing the angle relationships and the side relationships for similar triangles can be very useful in problem solving.

What Lies Ahead

Throughout the topic you will learn to

1. define similar triangles by investigating the special relationships between angles, and identify the corresponding sides and the corresponding angles
2. investigate and identify the special relationship between corresponding sides, and solve for an unknown side
3. solve practical problems involving an unknown side in similar triangles

Now that you know what to expect, turn the page to begin your study of similar triangles.

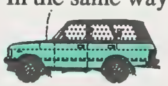
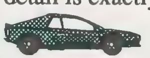
Exploring Topic 2

Activity 1

Define similar triangles by investigating the special relationships between angles, and identify the corresponding sides and the corresponding angles.

Enlarging or reducing a triangle gives other triangles that have the same shape.

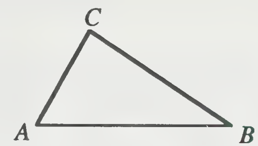
When you magnify an image using an overhead projector, the projection has the same shape as the original.

In the same way, when  is enlarged to , the size changes, but rest of the detail is exactly the same. Similarly, when  is reduced to , the size changes, but the rest of the detail is exactly the same. In each case the two recreational vehicles and the two cars are similar.

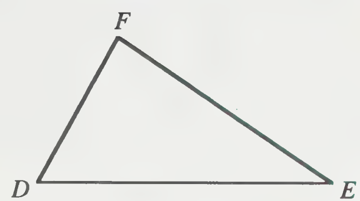
By measuring the angles of triangles that have the same shape, you will discover a particular relationship that defines **similar triangles**.

The questions that follow give you a chance to see a relationship for similar triangles.

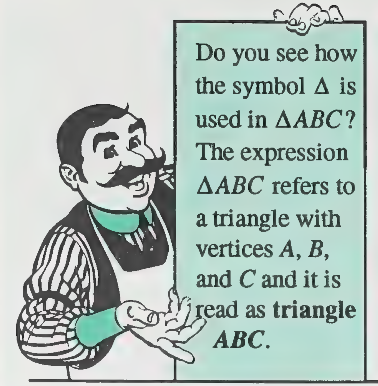
1. Note that ΔABC and ΔDEF have the same shape. Using a protractor, measure the angles of these triangles.



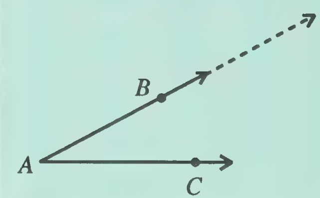
$\angle A =$
$\angle B =$
$\angle C =$



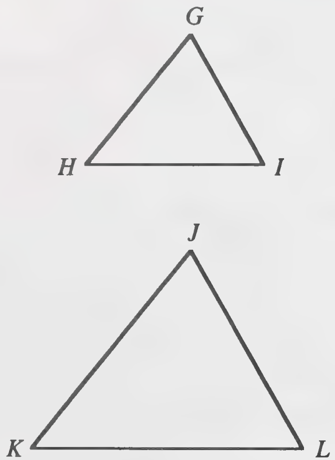
$\angle D =$
$\angle E =$
$\angle F =$



Sometimes extending an arm or ray of an angle makes it easier to measure accurately. Notice how $\overrightarrow{AB}$ is extended in the following angle.



2. Note that ΔGHI and ΔJKL have the same shape. Using a protractor, measure the angles of ΔGHI. For each of the three angles in ΔGHI, find the angles in ΔJKL that have the same measure.



For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

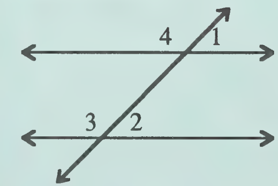
In the previous questions you found that for two triangles having the same shape, you could find pairs of angles with equal measures. Such pairs of angles are called **corresponding angles**. This leads to a definition of similar triangles.

Similar triangles are triangles which have corresponding angles of equal measure.

A special notation can be used to express similarity between triangles. Referring to the previous triangles in this activity, you can write $\Delta ABC \sim \Delta DEF$ and $\Delta GHI \sim \Delta JKL$. The symbol ~ means similar. In such similarity statements, letters in the same position must refer to corresponding angles. In other words, the vertices must be named in correct order. For example, in the statement $\Delta ABC \sim \Delta DEF$, $\angle B \cong \angle E$, $\angle A \cong \angle D$, and $\angle C \cong \angle F$. On the other hand, it would be incorrect to write $\Delta GHI \sim \Delta LJK$ because $\angle G \neq \angle L$, $\angle H \neq \angle J$, and $\angle I \neq \angle K$. So watch the ordering of the letters since the order used is really important.

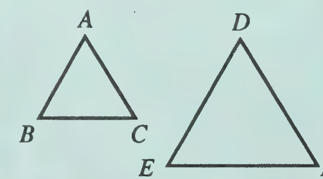
Corresponding angles have a different meaning when applied to parallel lines than when they are applied to similar triangles. This difference is shown as follows.

- Parallel lines



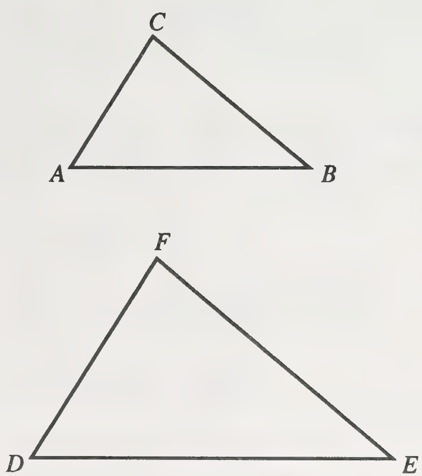
In this diagram, $\angle 1$ and $\angle 2$ are corresponding angles and $\angle 3$ and $\angle 4$ are corresponding angles.

- Similar triangles



In this diagram, $\angle A$ and $\angle D$ are corresponding angles, $\angle B$ and $\angle E$ are corresponding angles, and $\angle C$ and $\angle F$ are corresponding angles.

Corresponding angles have the same measure. For similar triangles there are also **corresponding sides**, but they are not necessarily the same length. Learn how to identify corresponding sides and then you can look for a relationship between them.



In the preceding diagram $\Delta ABC \sim \Delta DEF$. The following are pairs of corresponding sides.

- $\overline{AB}$ and $\overline{DE}$
- $\overline{BC}$ and $\overline{EF}$
- $\overline{CA}$ and $\overline{FD}$

Did you notice that $\overline{AB}$ is opposite $\angle C$ and $\overline{DE}$ is opposite $\angle F$?

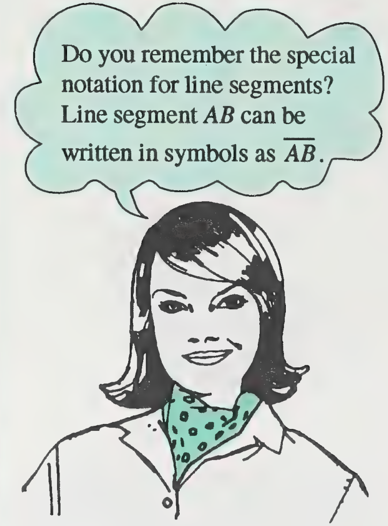
Did you notice that $\overline{BC}$ is opposite $\angle A$ and $\overline{EF}$ is opposite $\angle D$?

Did you notice that $\overline{CA}$ is opposite $\angle B$ and $\overline{FD}$ is opposite $\angle E$?

Corresponding sides are always opposite corresponding angles.

The sides that are opposite corresponding angles are called **corresponding sides.**

You should be able to list corresponding sides and corresponding angles from a similarity statement. This is shown in the example that follows.



Example 1

Suppose that $\Delta MNO \sim \Delta ABC$. List the corresponding angles and corresponding sides.

Solution:

If $\Delta MNO \sim \Delta ABC$, then

$\angle M$ corresponds to $\angle A$

$\overline{NO}$ corresponds to $\overline{BC}$

$\angle N$ corresponds to $\angle B$

$\overline{MO}$ corresponds to $\overline{AC}$

$\angle O$ corresponds to $\angle C$

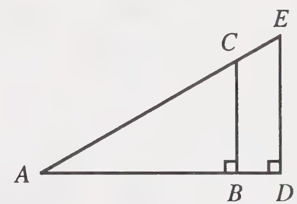
$\overline{MN}$ corresponds to $\overline{AB}$

Do you think that drawing and labelling two similar triangles would help you understand this better?

The following example shows how similar triangles can be identified.

Example 2

Name the similar triangles in the diagram. Give reasons for your answer.



Solution:

The similar traingles in the diagram are called ΔABC and ΔADE.

Since $\angle BAC$ and $\angle DAE$ are the same angle, $\angle BAC = \angle DAE$.

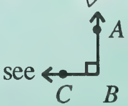
Since both $\angle ABC$ and $\angle ADE$ measure $90°$, $\angle ABC = \angle ADE$.

Since $\angle ACB$ and $\angle AED$ are third angles, $\angle ACB = \angle AED$.

Therefore, $\Delta ABC \sim \Delta ADE$ since corresponding angles have equal measure.

These symbols are used often. Make sure you know what each one means.

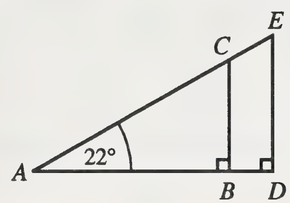
The symbol ~ means similar.
The symbol $\cong$ means congruent.
The symbol $\doteq$ means approximately equal.

When you see 

, the little square means that $\angle ABC = 90°$. Thus, it also means that $\angle ABC$ is a right angle.

When two angles of one triangle are equal in measure to two corresponding angles of another, the remaining angles in the triangles are called **third angles**. Therefore, third angles have the same measure.

Study this diagram which shows why third angles of two triangles are congruent.



If $\angle A = 22°$ and $\angle B = 90°$, then the measure of $\angle C$ is as follows:

$$\begin{aligned} \angle C &= 180° - (22° + 90°) \\ &= 180° - 112° \\ &= 68° \end{aligned}$$

If $\angle A = 22°$ and $\angle D = 90°$, then the measure of $\angle E$ is as follows:

$$\begin{aligned} \angle E &= 180° - (22° + 90°) \\ &= 180° - 112° \\ &= 68° \end{aligned}$$

Example 3

For the triangles in Example 2, list the corresponding sides.

Solution:

Since $\Delta ABC \sim \Delta ADE$, then

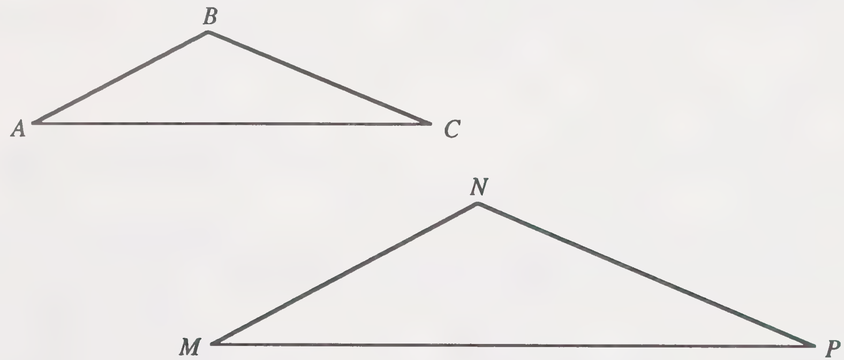
$\overline{AB}$ corresponds to $\overline{AD}$

$\overline{BC}$ corresponds to $\overline{DE}$

$\overline{AC}$ corresponds to $\overline{AE}$

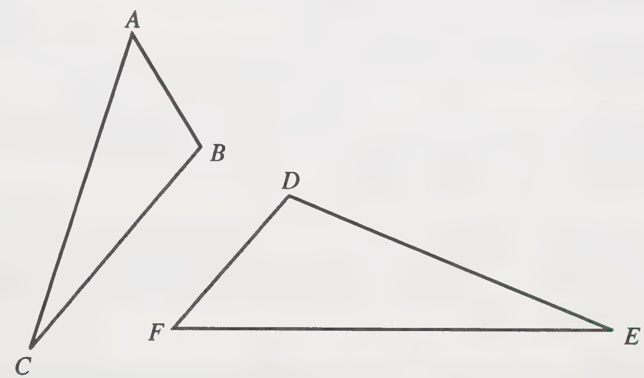
Now use this knowledge to help you do the questions that follow.

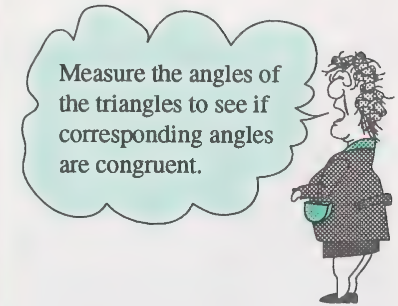
3. Compare ΔABC and ΔMNP. Assume that the angles of both triangles are drawn to the nearest degree. Are these triangles similar? Give reasons for your answer.

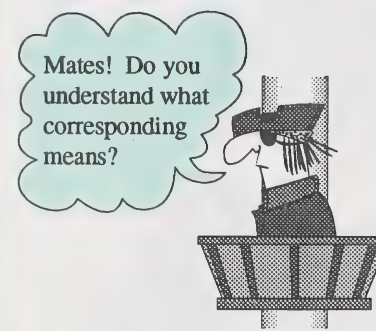


4. Note that $\Delta ABC \sim \Delta FDE$.

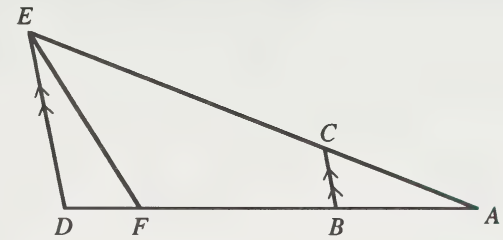
a. Name the corresponding angles.
b. Name the corresponding sides.



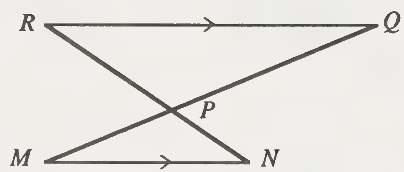




5. Which two triangles in the diagram are similar? Give reasons for your answer.



6. a. Name the similar triangles in the diagram. Give reasons for your answer. Notice that $\overline{RQ}$ is parallel to $\overline{MN}$.



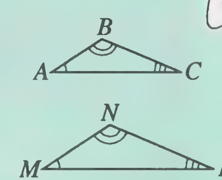
b. For the congruent triangles, list the corresponding sides.

For solutions to **Activity 1**, turn to **Appendix A, Topic 2**.

Hint: Use the fact that $\overline{DE} \parallel \overline{BC}$ for some of the angle relationships. One of the triangles in the congruency relationship is ΔABC.

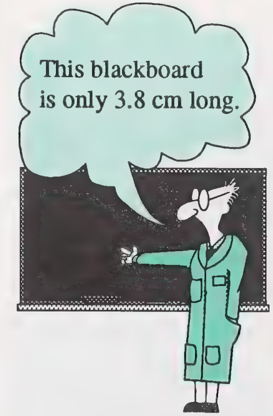
One common method of indicating that two angles are congruent is to mark the angles as shown.

For example, in ΔABC and ΔMNP the corresponding angles are congruent. Therefore, $\Delta ABC \sim \Delta MNP$.



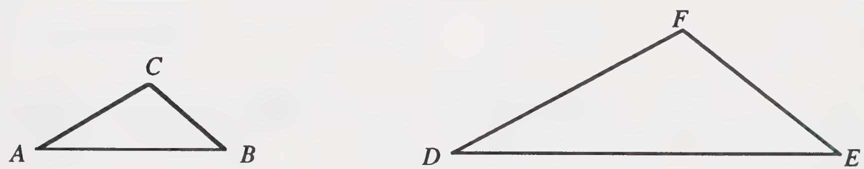
Activity 2

Investigate and identify the special relationship between corresponding sides, and solve for an unknown side.



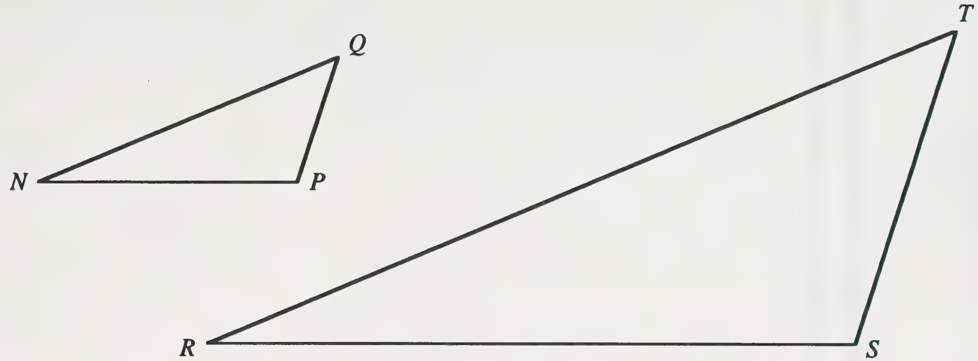
You have discovered the relationship between corresponding angles in similar triangles. In the questions that follow, you should be able to discover a relationship between corresponding sides of similar triangles.

1. Note that $\Delta ABC \sim \Delta DEF$. Use a centimetre ruler to measure the sides of these triangles; then give answers for the following to the nearest tenth of a centimetre.



$AB =$	$BC =$	$CA =$
$DE =$	$EF =$	$FD =$
$\frac{DE}{AB} =$	$\frac{EF}{BC} =$	$\frac{FD}{CA} =$

2. Note that $\Delta NPQ \sim \Delta RST$. You can check this by measuring and comparing the corresponding angles with your protractor. Then measure the sides and answer the following.



$NP =$	$PQ =$	$NQ =$
$RS =$	$ST =$	$RT =$
$\frac{NP}{RS} =$	$\frac{PQ}{ST} =$	$\frac{NQ}{RT} =$

3. For similar triangles, how do the quotients formed by dividing corresponding side lengths compare?

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 2**.

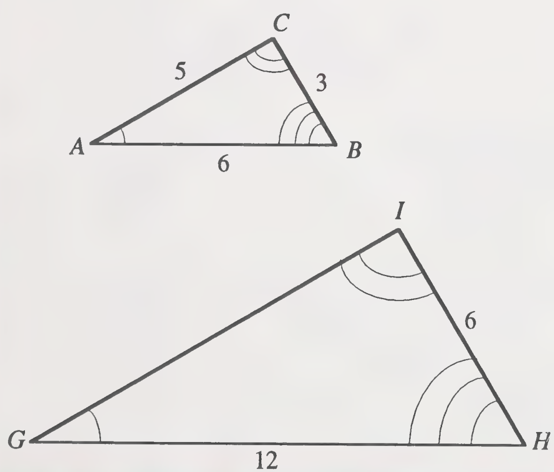
In the previous questions you discovered how corresponding sides are related. The relationship can be stated as follows:

For similar triangles, pairs of corresponding side lengths always form the same ratio.

Knowing this relationship for corresponding sides allows you to solve for any unknown length in a pair of similar triangles. This is shown in the following examples.

Example 4

Note that $\Delta ABC \sim \Delta GHI$.



What is the length of $\overline{GI}$?

Solution:

$$\frac{GH}{AB} = \frac{GI}{AC} \quad \text{(Corresponding sides form the same ratio.)}$$

$$\frac{12}{6} = \frac{GI}{5}$$

$$12 \times 5 = 6 \times GI$$

$$GI = \frac{\overset{2}{\cancel{12}} \times 5}{\underset{1}{\cancel{6}}}$$

$$GI = 2 \times 5$$

$$GI = 10$$

or

$$\frac{HI}{BC} = \frac{GI}{AC}$$

$$\frac{6}{3} = \frac{GI}{5}$$

$$6 \times 5 = 3 \times GI$$

$$GI = \frac{\overset{2}{\cancel{6}} \times 5}{\underset{1}{\cancel{3}}}$$

$$GI = 2 \times 5$$

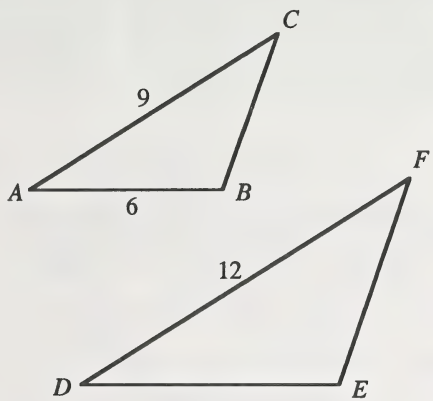
$$GI = 10$$

Did you notice in Example 4 that the corresponding sides in ΔGHI are **twice** those in ΔABC?

Do you remember this?
If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
This means you can cross-multiply and get the same product.

Example 5

If $\Delta ABC \sim \Delta DEF$, determine the value for DE.



Solution:

Let $DE = f$.

$\frac{9}{12} = \frac{6}{f}$ (Corresponding sides form the same ratio.)

$9f = 12 \times 6$

$f = \frac{12 \times 6}{9}$

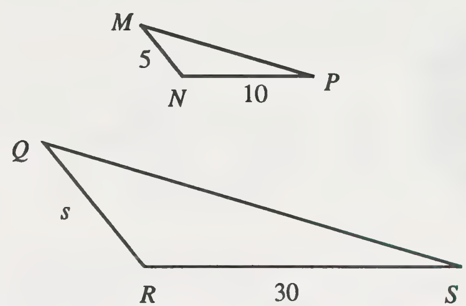
$= 8$

$\therefore DE = 8$

In the following questions you will use the relationship between sides of similar triangles.

Do at least three of the following questions.

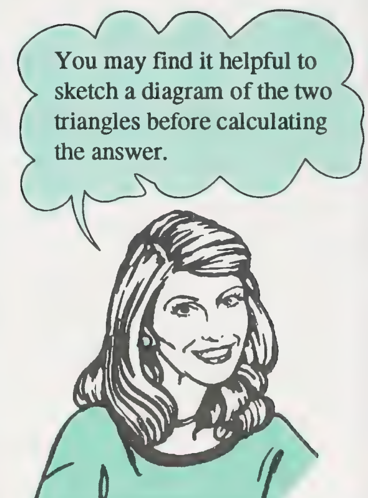
4. Given that $\Delta MNP \sim \Delta QRS$, find the value for QR.



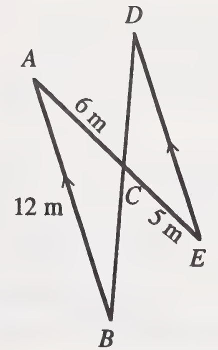
5. Given the following information, find UV. Round your answer to the nearest tenth. (Hint: Notice that ΔTUV is larger than ΔABC and that $\overline{AB}$ corresponds to $\overline{TU}$.)

$\Delta ABC \sim \Delta TUV$

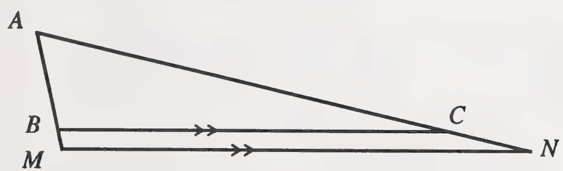
$AB = 4.0$ $TU = 6.0$
$AC = 5.0$
$BC = 3.0$



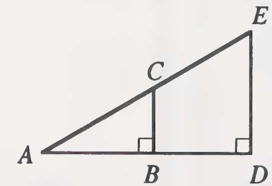
6. In the diagram $\overline{AB} \parallel \overline{ED}$ and certain side lengths are shown. Determine the length of DE.



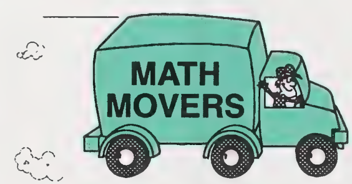
7. Refer to the diagram where $AB = 3\text{ m}$, $AM = 4\text{ m}$, and $MN = 16\text{ m}$. Calculate the length for BC.



8. From the diagram, determine the length for CE. Give your answer to the nearest tenth of a metre. Assume that $AB = 3.0\text{ m}$, $AD = 5.0\text{ m}$, and $AC = 3.3\text{ m}$. (Hint: First find the length of AE to begin your solution.)



For solutions to **Activity 2**, turn to **Appendix A, Topic 2**.



Activity 3

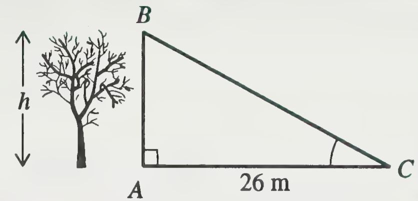
Solve practical problems involving an unknown side in similar triangles.

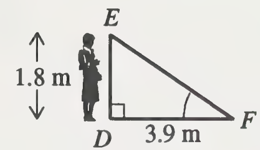
What you have learned about solving for unknown sides in similar triangles can be applied in practical problems. This is especially useful when actual measurements are inconvenient or impossible. The following examples show how your knowledge of similar triangles can be used to find distances indirectly. To find a distance **indirectly** means that actual measurement is not used.

Example 6

Carol was wondering about the height of a tree in her backyard. On a sunny day she found that the tree cast a 26 m shadow. At the same time she cast a shadow 3.9 m long. Carol's height is 1.8 m. What is the height of the tree?

Solution:





Since $\Delta ABC \sim \Delta DEF$, the corresponding sides of these similar triangles form the same ratio. This relationship can be set up to result in the following proportion.

$$\frac{h}{1.80} = \frac{26}{3.9}$$

$$h \times 3.9 = 26 \times 1.8$$

$$h = \frac{26 \times 1.8}{3.9}$$

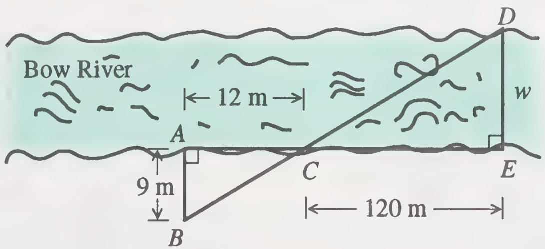
$$h = 12$$

Therefore, the height of the tree is 12 m.

The sun's rays are parallel so $\angle ACB$ and $\angle DFE$ would be corresponding and therefore congruent angles.

Example 7

The following diagram shows measurements made to indirectly determine the width of the Bow River at one spot. How wide is the river?



Solution:

Note that $\Delta ABC \sim \Delta EDC$.
Let w be the width of the river in metres. Since corresponding sides form the same ratio, you get a proportion as shown.

$$\frac{9}{w} = \frac{12}{120}$$
$$9 \times 120 = w \times 12$$
$$w = \frac{9 \times 120}{12}$$
$$w = 90$$

The width of the Bow River is 90 m at that particular spot.

In the diagram look for the corresponding angles in the two triangles.

Since they both measure 90°, $\angle BAC = \angle DEC$.

Since they are vertically opposite angles, $\angle ACB = \angle ECD$.

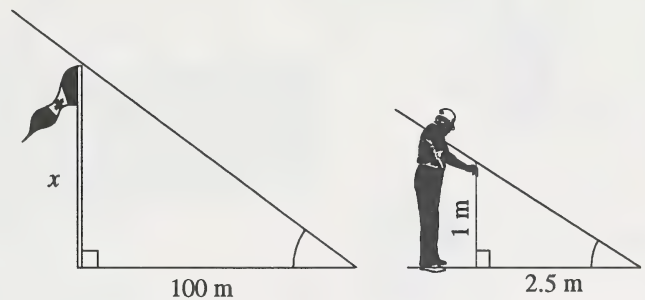
Since they are third angles, $\angle ABC = \angle EDC$.

Since the corresponding angles of the two triangles are congruent, the two triangles must be similar. Furthermore, $\overline{AB}$ corresponds to $\overline{ED}$, $\overline{AC}$ corresponds to $\overline{EC}$, and $\overline{BC}$ corresponds to $\overline{DC}$ because the sides opposite the corresponding angles are known to be corresponding sides.

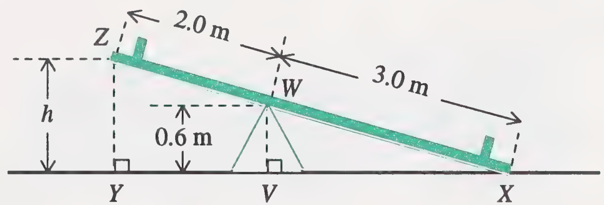
To solve practical problems, it is usually helpful if you sketch a diagram to relate similar triangles to the real situation. Although your sketches do not have to be drawn to scale, you should be able to show corresponding angles. Place known measurements on your diagram and identify the unknown side. Always check your answers to see if they are reasonable.

Do at least three of the following questions.

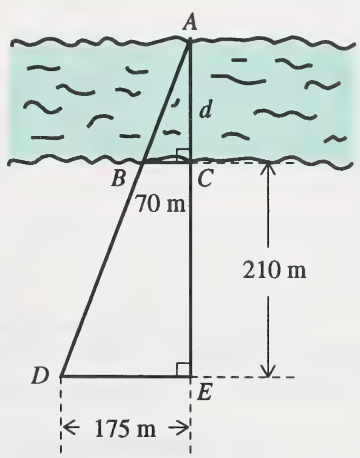
1. Alberto finds that the shadow cast by a flagpole is 100 m long. In order to determine the height of the pole, Alberto holds a metre stick upright with one end on the ground. He measures the stick's shadow to be 2.5 m long. How high is the flagpole?



2. An adjustable seesaw was left with one end resting on the ground. Use the information in the following diagram to find how high the other end is above the ground.

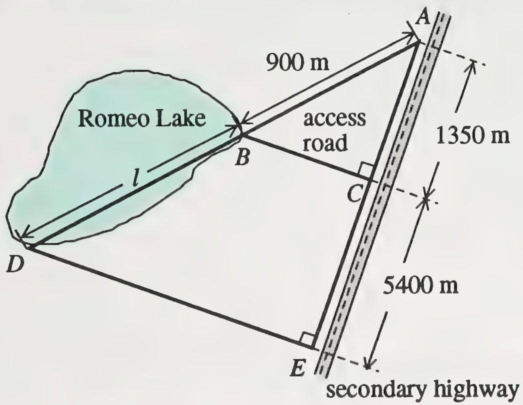


3. Use the information in the diagram that follows to find the width of the river.



Use this clue.
Identify similar triangles to show that $\dfrac{d}{d+210} = \dfrac{70}{175}$ and then solve for d.

4. Determine the length of Romeo Lake from the information in the diagram.

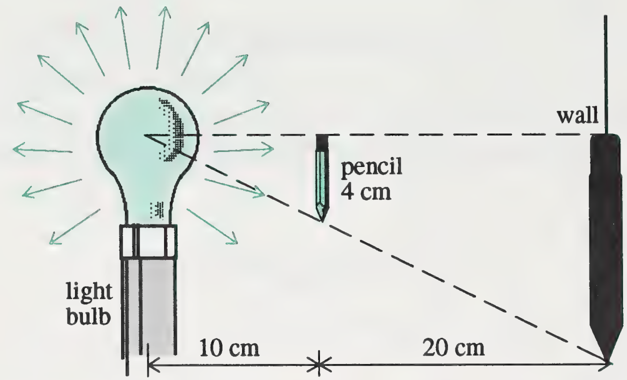


Here's a clue for question 4!
Identify the similar triangles and use the corresponding sides to get ratios to show the following:

$$\frac{l+900}{900}=\frac{5400+1350}{1350}$$

Then solve for l.

5. A 4 cm pencil is suspended 10 cm from a point source of light so that its total shadow falls on a wall. The wall is 20 cm from the pencil. What is the length of the shadow? The point of the light source and the top of the pencil are on the same horizontal plane.



For solutions to **Activity 3**, turn to **Appendix A, Topic 2.**

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

$\left.\right\}$ You may decide to do both.

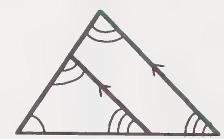
Extra Help

Difficulties with this topic may stem from not seeing which triangles are similar. In most situations similar triangles will be apparent in one of the following three forms.

Case I: The triangles are separate.

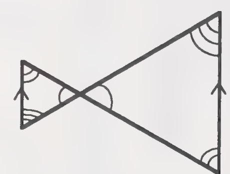
In Case I you must be sure about which angles correspond, and then you know the corresponding sides.

Case II: The triangles overlap.



In Case II the angles along a shared side are equal in measure.

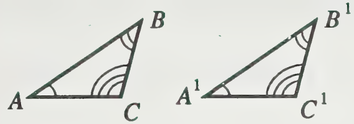
Case III: The triangles have vertically opposite angles.



In Case III the angles seem to flip around. The corresponding angles are across the point of intersection from each other. From the angle correspondence, follow the side correspondence.

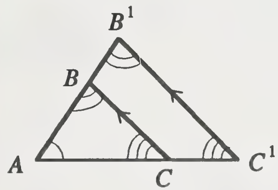
Now you can see the side ratios for each case.

Case I



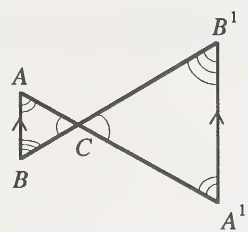
$$\frac{AC}{A^1C^1}=\frac{AB}{A^1B^1}=\frac{BC}{B^1C^1}$$

Case II



$$\frac{AB}{AB^1}=\frac{BC}{B^1C^1}=\frac{AC}{AC^1}$$

Case III



$$\frac{AB}{A^1B^1}=\frac{BC}{B^1C}=\frac{AC}{A^1C}$$

The superscripts are used to make the angle and side correspondence easier to see. These superscripts occur when points or vertices are labelled or named with a number above and to the right of the letter.

Note that angles labelled with the same letter are equal in measure.

Now you should be able to apply your knowledge of side ratios in the following questions.

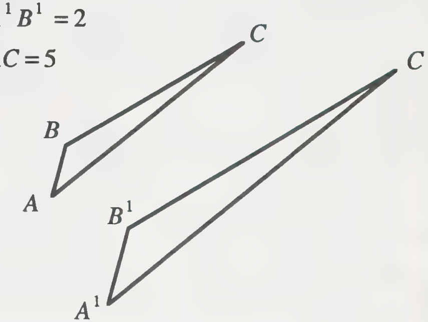
1. Use the following to calculate A^1C^1.

$\Delta ABC \sim \Delta A^1B^1C^1$

$AB = 1$

$A^1B^1 = 2$

$AC = 5$



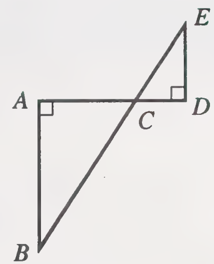
2. Use the following to calculate DC.

$\Delta ABC \sim \Delta DEC$

$AB = 10$

$DE = 5$

$AC = 8$



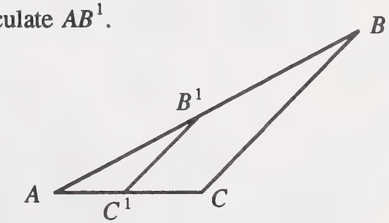
3. Use the following to calculate AB^1.

$\Delta ABC \sim \Delta AB^1C^1$

$B^1C^1 = 7$

$BC = 14$

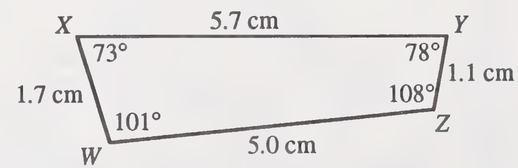
$AB = 22$

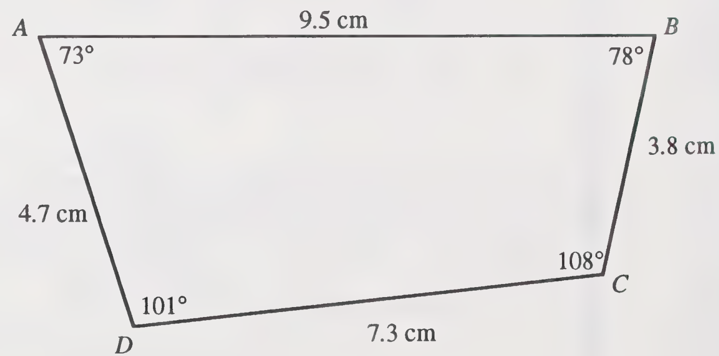


For solutions to **Extra Help**, turn to **Appendix A, Topic 2**.

Extensions

You have studied the concept of similarity for triangles. You could think of a triangle that is similar to another triangle as being a magnification or reduction of the other. That means corresponding sides form a constant ratio with each other when corresponding angles are equal in measure. Does this hold for a quadrilateral?





Use the diagram of quadrilaterals *WXYZ* and *ABCD* to answer the following questions.

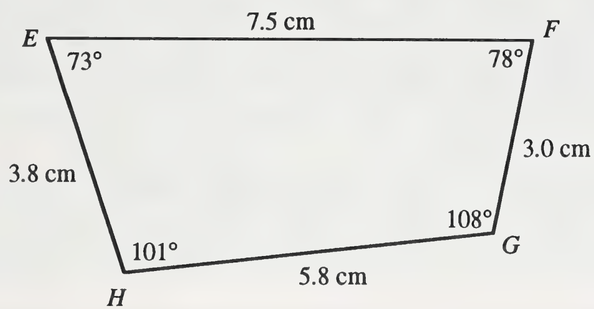
1. Tell which angles correspond allowing for some measurement error.

2. Calculate the ratios made by corresponding sides.

3. Are the quadrilaterals similar? Give reasons for your answer.

For solutions to **Extensions**, turn to **Appendix A, Topic 2**.

In the quadrilaterals you just studied, you found that just because there were corresponding angles of equal measure, the corresponding sides did not necessarily form the same ratio. Therefore, even if corresponding angles are the same in two quadrilaterals, one quadrilateral may not really be a magnification or reduction of the other.



The preceding quadrilateral *EFGH* really is a reduction of quadrilateral *ABCD* from the previous diagram. In fact, it could be made by using the reduction feature of a photocopying machine. The next questions will ask you to compare features of these two figures.

Use the diagrams of quadrilaterals *ABCD* and *EFGH* in this section to answer the following questions.

4. Tell which angles correspond allowing for some measurement error.

5. Calculate the ratios made by corresponding sides. Round your answers to two decimal places.

6. Are the quadrilaterals similar? Give reasons for your answer.

For solutions to **Extensions**, turn to **Appendix A, Topic 2**.

For triangles to be similar, all you have to know is that corresponding angles are equal in measure. Then it follows that corresponding sides will form the same ratio. However, you found that for quadrilaterals the conditions for similarity are more strict.

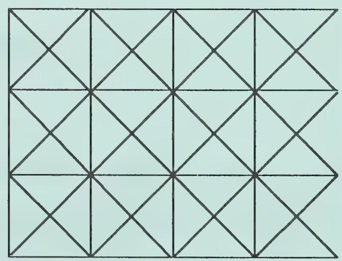
Quadrilaterals are similar when both of the following conditions are satisfied.

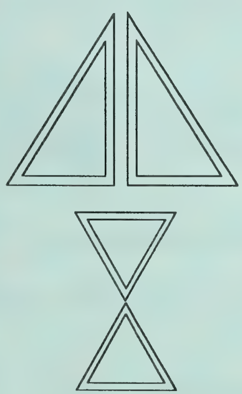
- Corresponding angles are equal in measure.
- Corresponding sides form the same ratio.

Topic 3 Congruent Triangles

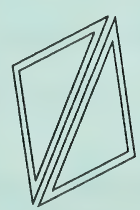
Introduction

Congruent triangles are not only the same shape, but they are also the same size.





In this topic you will learn how to tell when triangles are congruent. Then you will solve problems using congruent triangles.



What Lies Ahead

Throughout the topic you will learn to

1. define congruency as applied to triangles, and identify corresponding parts of congruent triangles
2. identify the conditions for congruence of two triangles, and solve problems involving congruent triangles

Now that you know what to expect, turn the page to begin your study of congruent triangles.

Exploring Topic 3

Activity 1

Define congruency as applied to triangles, and identify corresponding parts of congruent triangles.

Some triangles can be made to fit over top of each other. The idea of fit has a lot to do with congruency. The activity that follows asks you to pick triangles that fit together. You will need to use the page of triangles located in **Appendix B**. Cut out the triangles and keep them to use as needed throughout this topic.

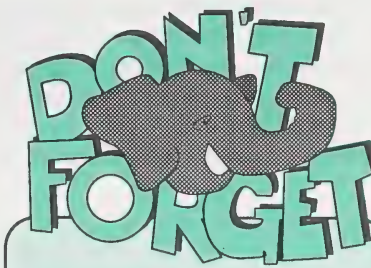
1. Use the triangles you have cut out from **Appendix B**. Tell which triangles can be made to fit over top of each other. For example, $\Delta 3$ and $\Delta 5$ fit over top of each other.

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

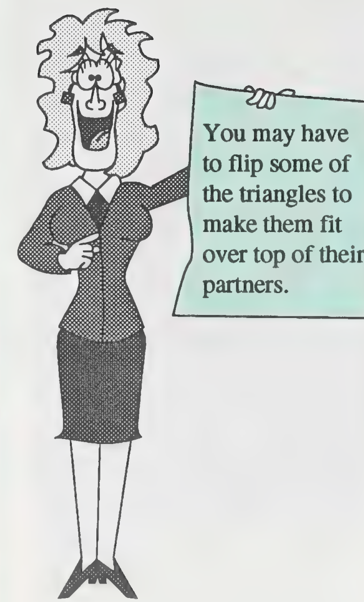
The triangles that you found to fit over top of each other are congruent triangles. Congruent triangles have the same shape and size. In other words, congruent triangles are similar triangles which are of the same size.

Congruent triangles are same-size similar triangles.

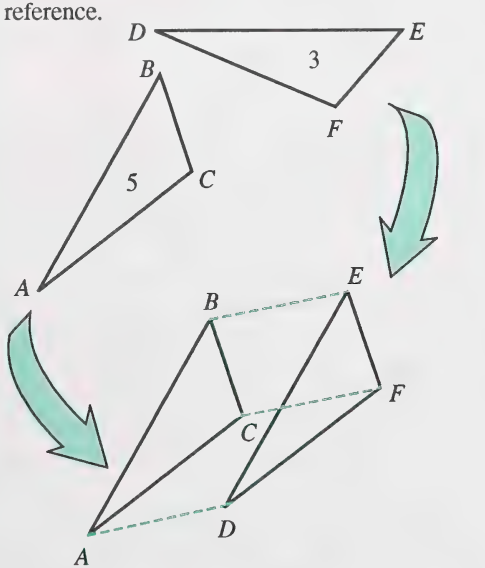
From this definition you can see that congruent triangles not only have corresponding angles of equal measure, but they also have corresponding sides of equal length.



Recall: For similar triangles the corresponding sides result in the same ratio.



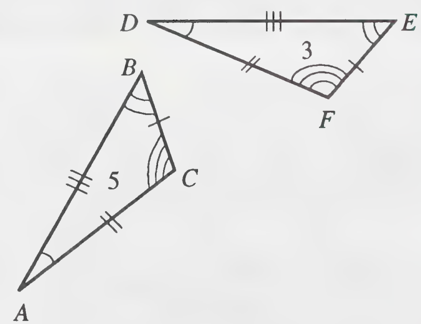
How can you decide which sides and which angles correspond? Look at $\Delta 3$ and $\Delta 5$ from the previous question. When the cutouts are placed over top of each other, the corresponding parts of the triangles are on top of each other. The vertices are labelled here for easier reference.



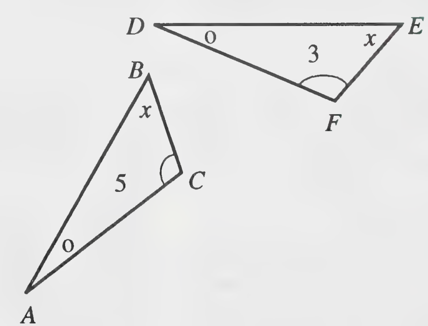
From the way the triangles fit, you can see that

$\angle A$ corresponds to $\angle D$
$\angle B$ corresponds to $\angle E$
$\angle C$ corresponds to $\angle F$
$\overline{AB}$ corresponds to $\overline{DE}$
$\overline{BC}$ corresponds to $\overline{EF}$
$\overline{AC}$ corresponds to $\overline{DF}$

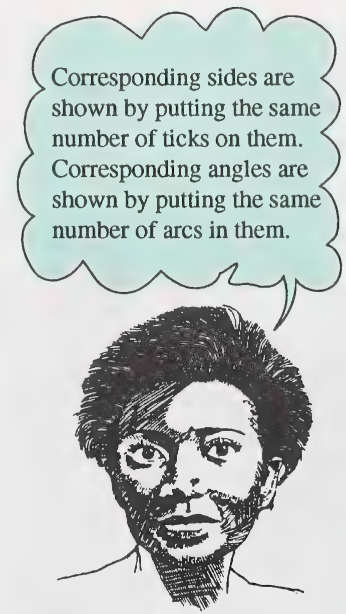
This correspondence can be indicated on the triangles like this.



In any diagram, angles marked the same way have the same measure, and sides marked the same way have the same length. Other symbols can be used for the angles as long as you use the **same symbol** for corresponding angles. Notice the symbols that are used in the following.



To indicate the congruence of $\Delta 5$ and $\Delta 3$, you could write that $\Delta ABC \cong \Delta DEF$.

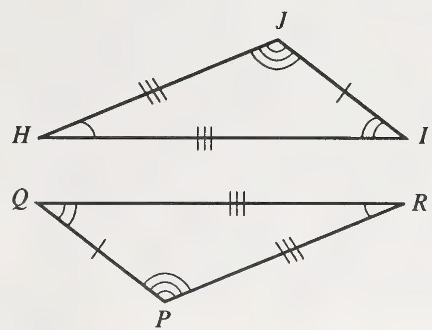


When using these symbols, letters in the same position must refer to corresponding angles. The symbol $\cong$ means **congruent**.

Now you should be able to identify corresponding parts of congruent triangles from diagrams. If the triangles are congruent, you can use the symbol $\cong$ to make a congruency statement.

Example 1

- List the corresponding angles and the corresponding sides.



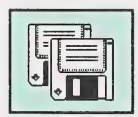
Solution:

$\angle H \leftrightarrow \angle R$	$\overline{HI} \leftrightarrow \overline{RQ}$
$\angle I \leftrightarrow \angle Q$	$\overline{IJ} \leftrightarrow \overline{QP}$
$\angle J \leftrightarrow \angle P$	$\overline{HJ} \leftrightarrow \overline{RP}$

- Complete the statement $\Delta HIJ \cong$ __________ .

Solution:

Letters for corresponding angles are put in the same position, so $\Delta HIJ \cong$ $\underline{\Delta RQP}$.

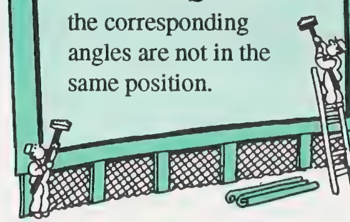


For a better understanding of corresponding parts of congruent triangles you may wish to use the diskette entitled *Geometry – Congruent Triangles,*[1] Lesson 1.

[1] *Geometry – Congruent Triangles* is a title of Scott, Foresman & Company.

Put these items in your memory bank.

- *JI* names the line segment $\overline{JI}$.
- *IJ* also names the line segment $\overline{JI}$.
- The symbol $\longleftrightarrow$ means **corresponds to.**
- You cannot use $\Delta HIJ \cong \Delta RPQ$ for $\Delta HIJ \cong \Delta RQP$ because the corresponding angles are not in the same position.

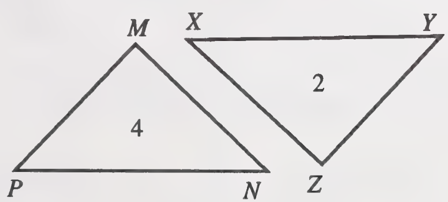


In these questions you can use your new insights about congruency. Do at least the odd-numbered questions. Some of the triangle cutouts from **Appendix B** are needed for questions 4, 5, and 6.

2. What are congruent triangles?

3. For congruent triangles, how do corresponding sides compare?

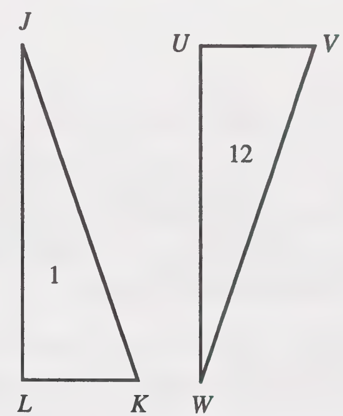
4. Place the $\Delta 4$ cutout on $\Delta 4$ in the following diagram. Label the angles in the same way. Place the $\Delta 2$ cutout on $\Delta 2$ in the following diagram. Label the angles in the same way. Now fit the cutouts over each other again. Observe which sides and angles are on top of each other. These angles and sides are the corresponding parts of $\Delta 4$ and $\Delta 2$.



   a. List the corresponding angles and sides for these triangles.

   b. Complete the congruency statement $\Delta MNP \cong$ __________ .

5. Place the $\Delta 1$ cutout on $\Delta 1$ in the following diagram. Label the angles in the same way. Place the $\Delta 12$ cutout on $\Delta 12$ in the following diagram. Label the angles in the same way. Now fit the cutouts over each other again. You will have to flip one of these cutouts to make them fit. Observe which sides and angles are on top of each other. These angles and sides are the corresponding parts of $\Delta 1$ and $\Delta 12$.



   a. List the corresponding angles and sides for the triangles.

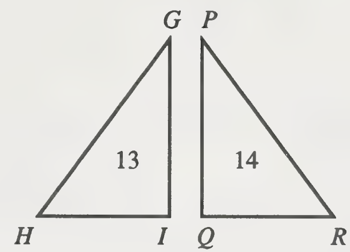
   b. Complete the congruency statement $\Delta JKL \cong$ __________ .

You may have to flip one of the cutouts in order that both triangles fit. Label both sides of the flipped cutouts to avoid confusion.

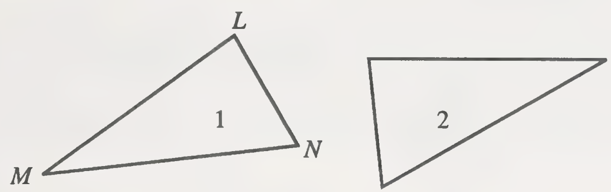
6. You have seen that $\Delta 13$ and $\Delta 14$ are congruent.

   a. On the following diagram, mark the angles and sides to show corresponding pairs.

   b. $\Delta GHI \cong$ ____________



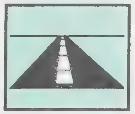
7. Label the angles in $\Delta 2$ in order to make the congruency statement $\Delta LMN \cong \Delta ABC$ apply to the two congruent triangles that follow.



For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

Remember to match the same-size angles and the same-length sides.

Activity 2



Identify the conditions for congruence of two triangles, and solve problems involving congruent triangles.

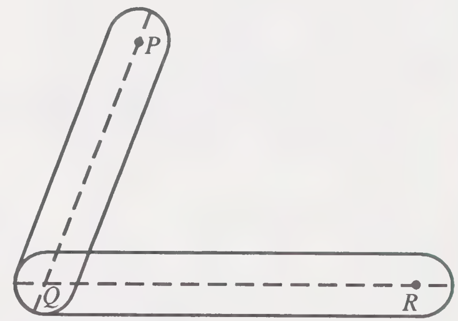
You have seen that congruent triangles have corresponding angles of the same measure and corresponding sides of the same length. However, you do not need to know all six equalities to conclude that two triangles are congruent. The following questions will help you discover how much information you need to be given in order to know all about a triangle.

For these questions you will need to use the figures on the last two pages of **Appendix B**.

1. a. Cut out Sides 1, 2, and 3 from **Appendix B** and make a triangle using these three sides. Join the sides so that the dots coincide. You may want to use straight pins to keep the dots in place.

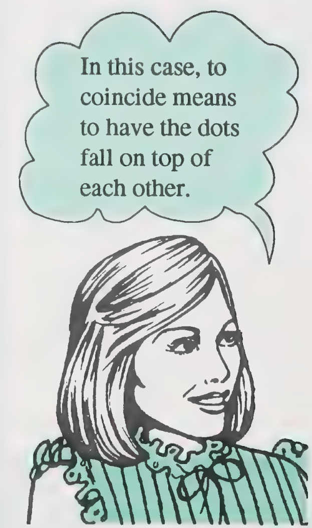
   b. Using the same three sides, can you make any different triangles? If so, build them.

2. Build a triangle given two sides and the angle that they form. Here $\overline{PQ}$, $\overline{QR}$, and $\angle PQR$ are given.

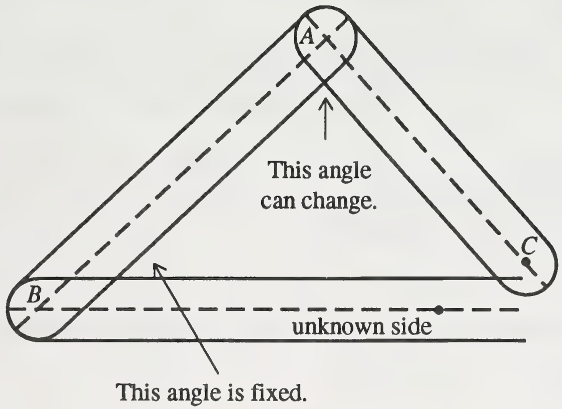


   a. Using the pattern provided in **Appendix B,** cut out a side of appropriate length and join P and R. Have the centre line go over P and R.

   b. Are there any different triangles you can build so that it includes $\overline{PQ}$, $\overline{QR}$, and $\angle PQR$? If so, build them.



3. Build a triangle given two sides and one of the angles formed with the unknown side. Here $\overline{AB}$, $\overline{AC}$, and $\angle ABC$ are given.

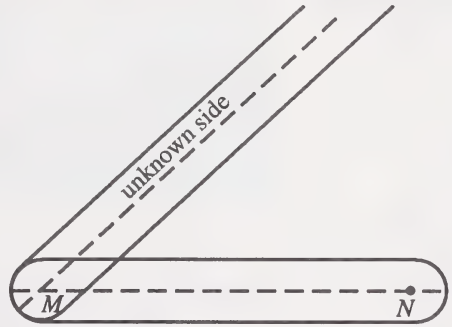


a. Cut out Side 4 from **Appendix B** and place it on the preceding diagram so that the dots marked *A* coincide and dot *C* falls on the centre line of the unknown side.

b. Is there any different triangle you can build so that it includes $\overline{AB}$, $\overline{AC}$, and $\angle ABC$? If so, build it.

In question 3, point *A* is fixed, but point *C* can be placed at two positions on the unknown side such that two different triangles may result.

4. Build a triangle given two angles and the side between their vertices. Here $\overline{MN}$, $\angle M$, and $\angle N$ are given.



a. Cut out $\angle N$ which is found on the last page of **Appendix B**. Then place $\angle N$ on the given diagram so that the dots marked M and the dots marked N coincide. Place a dot at the intersection of the centre lines of the sides that cross. Label this dot L.

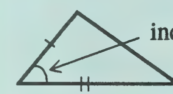
b. Is there any different triangle you can build so that it includes $\overline{MN}$, $\angle M$, and $\angle N$? If so, build it.

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 3**.

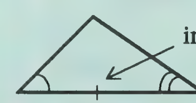
From the questions, you found that you did not need to be given all the angles and sides in order to build a specific triangle. From this you can conclude that any one of the following conditions is enough for congruence between triangles.

- The corresponding sides are equal in measure (SSS).
- Two of the corresponding sides and the angle they determine are equal in measure (SAS).
- Two of the corresponding angles and the side between their vertices are equal in measure (ASA).

For SAS you must have the included angle. The included angle is the angle between the two known sides.



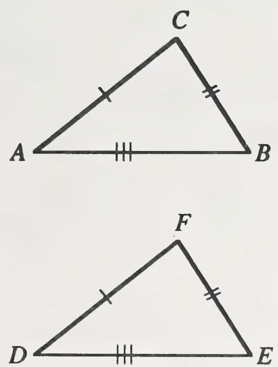
For ASA you must have the included side. The included side is the side that has the two known angles at the endpoints.



You can show that two triangles are congruent by demonstrating that one of SSS, SAS, or ASA has been met. This is shown in the following examples.

Example 2

Show that $\Delta ABC \cong \Delta DEF$.



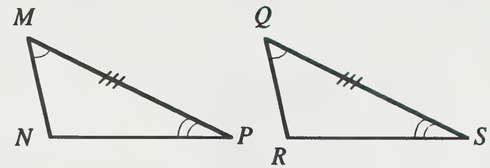
Solution:

$AB = DE$	given (S)
$BC = EF$	given (S)
$AC = DF$	given (S)

$\therefore \Delta ABC \cong \Delta DEF$ (SSS)

Example 3

Show that $\Delta MNP \cong \Delta QRS$.

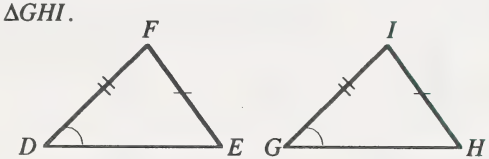


Solution:

$\angle M = \angle Q$	given (A)
$MP = QS$	given (S)
$\angle P = \angle S$	given (A)

$\therefore \Delta MNP \cong \Delta QRS$ (ASA)

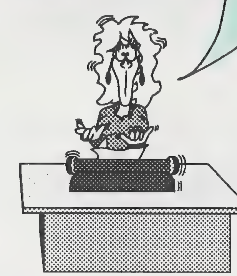
Sometimes it may look as if two triangles are congruent by SAS, but in fact SAS does not hold. Consider, for example, ΔDEF and ΔGHI.



You cannot conclude that these two triangles are congruent because the angle given is **not the included angle**. Recall that for a SAS congruency to exist, the angle given must be **between** the two given sides.

Memorize these abbreviations and their meanings.

- SSS means Side Side Side.
- ASA means Angle Side Angle.
- SAS means Side Angle Side.

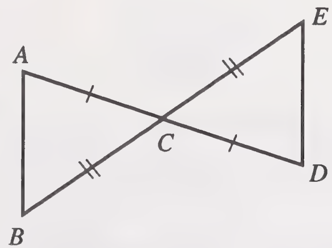


Given information is information that is supplied directly in the problem or in the diagram.

There are cases in which all the information required to show congruency is not given directly. You may need to use the fact that sides are shared or that angles have certain relationships. This is shown in the examples that follow.

Example 4

Show that $\Delta ABC \cong \Delta DEC$.



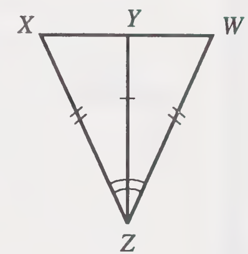
Solution:

$AC = DC$	given (S)
$\angle ACB = \angle DCE$	vertically opposite angles (A)
$BC = EC$	given (S)

$\therefore \Delta ABC \cong \Delta DEC$ (SAS)

Example 5

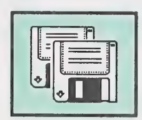
Show that $\Delta XYZ \cong \Delta WYZ$.



Solution:

$XZ = WZ$	given (S)
$\angle XZY = \angle WZY$	given (A)
$YZ = YZ$	same side (S)

A side which is common to two triangles is equal to itself in measure. Thus, $\Delta XYZ \cong \Delta WYZ$ because $\angle XZY$ and $\angle WZY$ are the included angles and an SAS congruency has been shown.



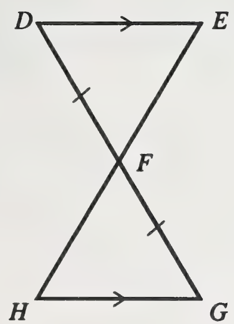
For additional practice with SSS, SAS, and ASA, you may use the diskette entitled *Geometry – Congruent Triangles,*[1] Lesson 3.

[1] *Geometry – Congruent Triangles* is a title of Scott, Foresman & Company.

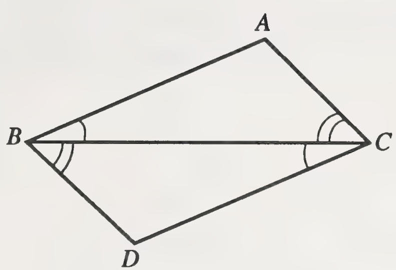
In the questions that follow, apply your understanding of congruent triangles.

Do at least the odd-numbered questions.

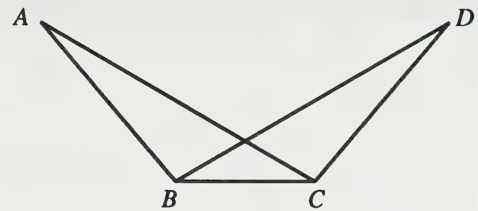
5. Suppose $EF = 2$. Show that $FH = 2$.



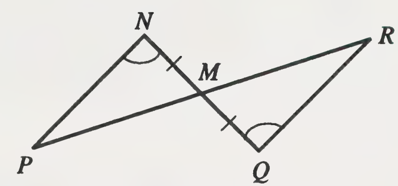
6. Show that $\Delta ABC \cong \Delta DCB$.



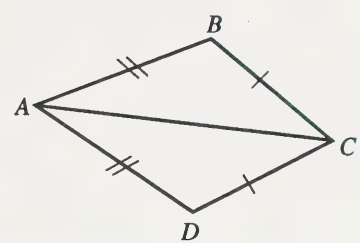
7. Show that $\Delta ABC \cong \Delta DCB$.
Assume $\angle ABC = \angle DCB$ and $AB = DC$.



8. Show that $\Delta MNP \cong \Delta MQR$.



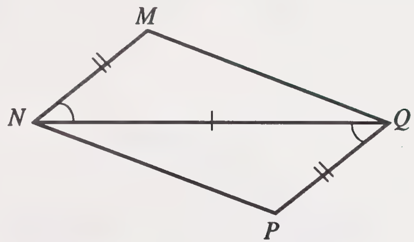
9. Show that $\Delta ABC \cong \Delta ADC$.



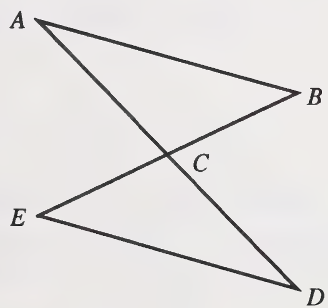
Hint: In question 5, first show that $\Delta DEF \cong \Delta GHF$. This will help with the solution.

You should know that symbols such as $\xrightarrow{}$ $\xrightarrow{}$ on the diagram mean that the two line segments are parallel.

10. Give the length of NP if $QM = 4$.

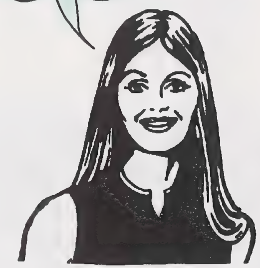


11. In the following diagram, $\angle ACB = 110°$, $AC = DC$, $BC = EC$, and $\angle CBA = 40°$. What is the measure of $\angle CED$? Give reasons for your answer.



For solutions to **Activity 2**, turn to **Appendix A**, **Topic 3**.

Hint: In question 10, show that $\Delta MNQ \cong \Delta PQN$. Use substitution to find the measures of corresponding parts.



Hint: In question 11, look for congruent triangles. This will give you corresponding angles that are equal in measure.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

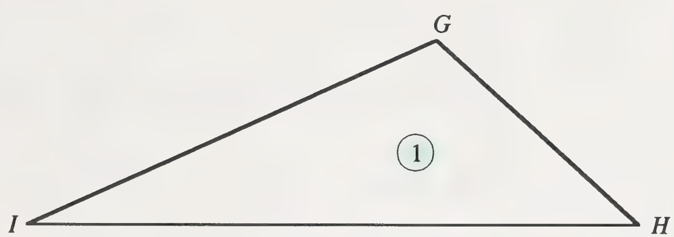
Extra Help

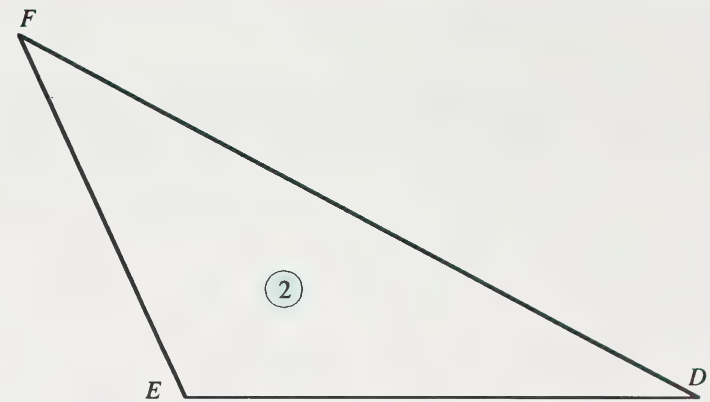
It may help you to take another look at triangle congruency. You can think of congruency as two triangles being able to fit over top of each other.

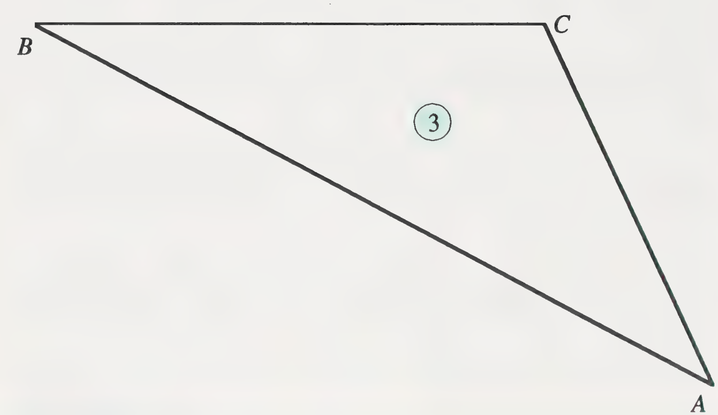
Two triangles are congruent when the following two conditions are met.

- Corresponding angles are equal.
- Corresponding sides are equal.

The following questions ask you to use these conditions to determine congruency in triangles. Use the next three figures to answer the questions that follow.







1. Measure the angles and sides of the three triangles.

2. From your measurements determine which triangles are congruent. Allow for some measurement error.

3. Show which angles and sides correspond for the congruent triangles.

4. Complete the following congruency statement.

$\Delta ABC \cong \Delta$ ________

For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.

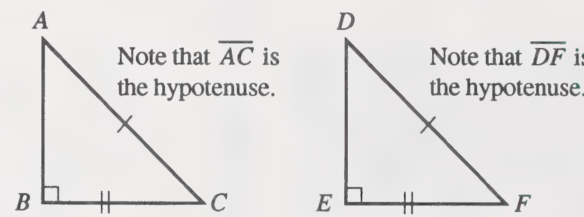
If you are confused and not too sure about how to answer the questions, check the answers in **Appendix A**, and then try the questions again.

Extensions

You have found that two triangles are congruent when a congruency condition (SSS, SAS, or ASA) has been met. There is another such condition that applies to two right triangles.

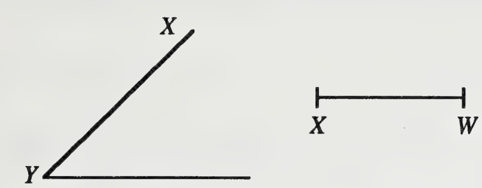
The HL condition requires that the hypotenuses in both triangles are equal in length and one leg of one triangle is equal in length to the corresponding leg of the other triangle.

Two right triangles are congruent when HL holds. For example, $\Delta ABC \cong \Delta DEF$ by HL.

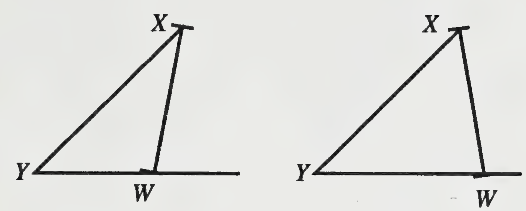


Notice that two sides and an angle, which is not the included angle, are given. If the given angle is a right angle, it does not have to be the included angle. That is why HL applies only to right triangles.

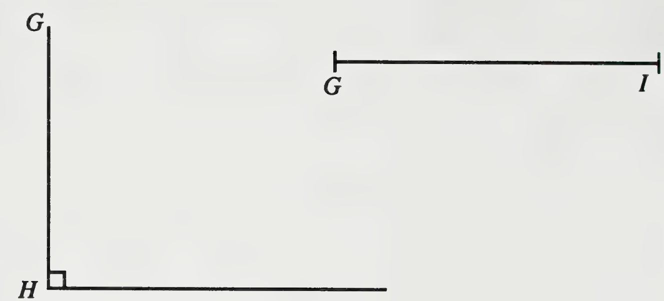
Suppose you were given $\angle Y$, side XY, and side XW, and you want to build a triangle.



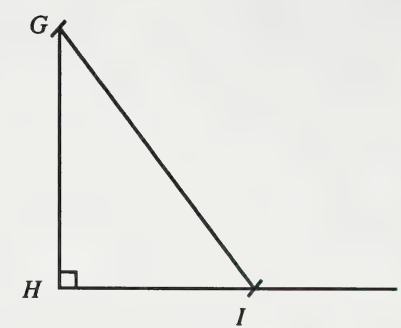
The triangle could be completed in these two ways.



Compare this with the situation in which one leg $\overline{GH}$ and a right angle H are given. There is only one way to complete the triangle with hypotenuse $\overline{GI}$.



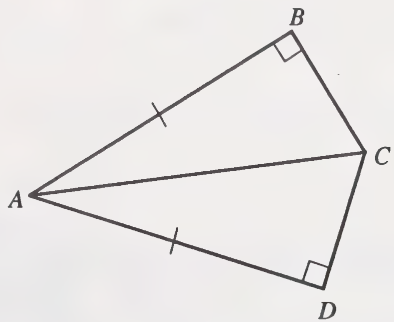
The completed triangle is shown as follows.



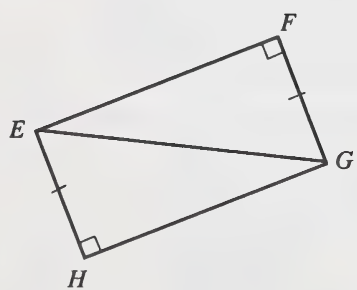
This demonstrates that the condition HL is enough for congruency in right triangles.

Now try some questions using your knowledge of HL.

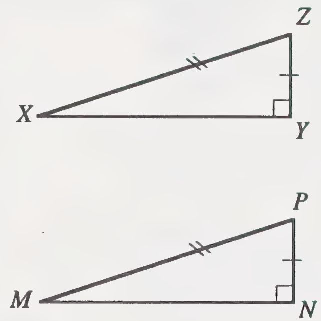
1. Show that $\Delta ABC \cong \Delta ADC$.



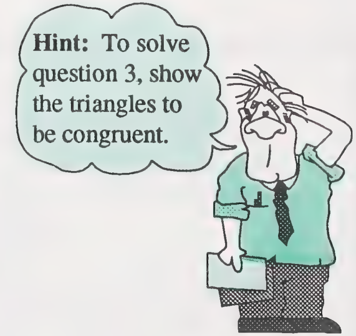
2. Show that $\Delta EFG \cong \Delta GHE$.

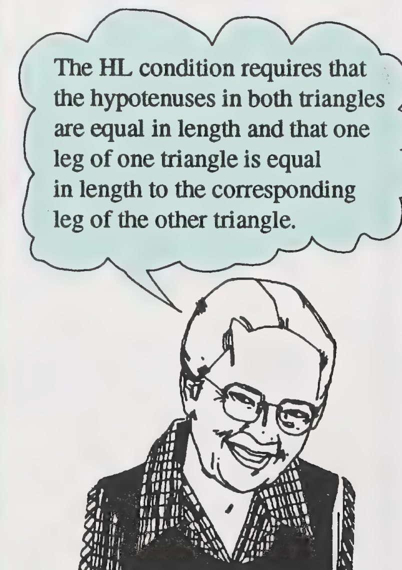


3. Show that $XY = MN$.



For solutions to **Extensions**, turn to **Appendix A, Topic 3**.





Topic 4 Constructing Triangles

Introduction

When constructing a building, certain equipment and tools are needed. When constructing triangles, you can use only a compass and a straightedge. By constructing triangles, you will be able to see that one of the properties (SSS, ASA, or SAS) is enough to show congruency.

What Lies Ahead

Throughout the topic you will learn to

1. construct a triangle given three sides
2. construct a triangle given two sides and the included angle
3. construct a triangle given two angles and the included side
4. construct a triangle congruent to a given triangle

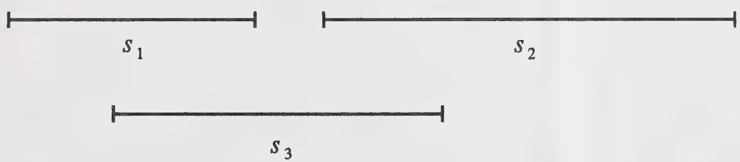
Now that you know what to expect, turn the page to begin your study of constructing triangles.

Exploring Topic 4

Activity 1

Construct a triangle given three sides.

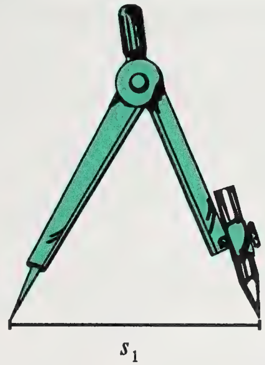
By knowing the lengths of the three sides of a triangle, you can construct it. This is shown in the following steps.



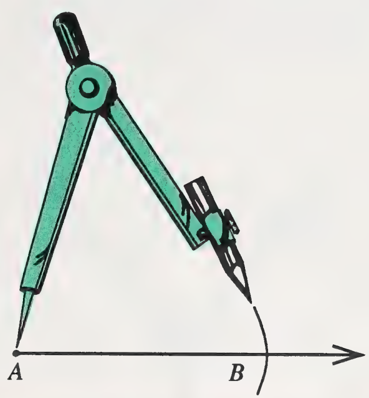
The steps for construction are as follows.

Step 1: Draw a ray longer than s_1 and label a point A on the left end of this ray.

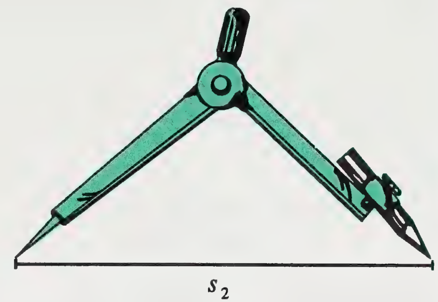
Step 2: Set your compass so that the point and the pencil are on the endpoints of s_1.



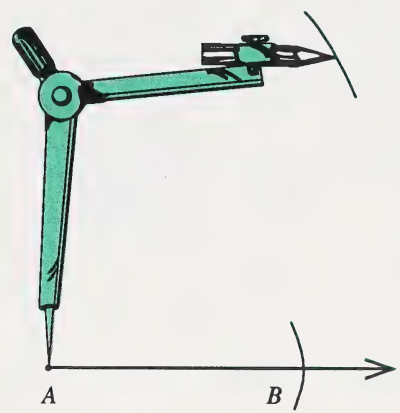
Step 3: Keeping the same setting, place the compass point at A and draw an arc across the ray you drew previously. Label the intersection as point B.

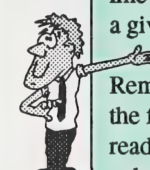


Step 4: Set your compass so that the point and the pencil are on the endpoints of s_2.



Step 5: Keeping the same setting, place the compass point at A and draw an arc above $\overline{AB}$ as shown.

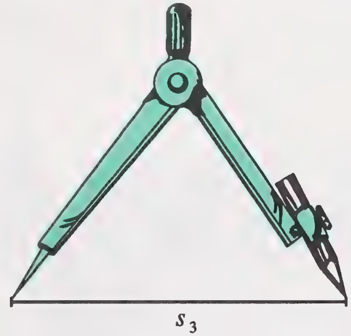




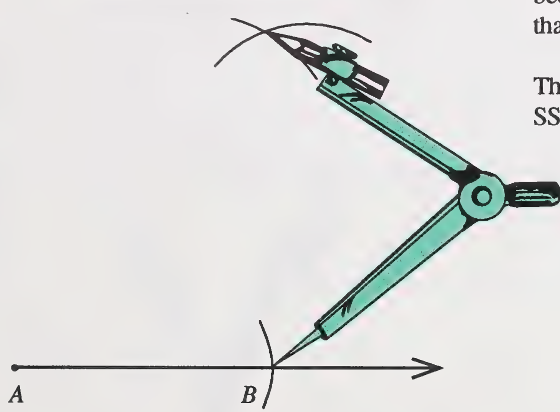
The first three steps result in the construction of a line segment congruent to a given line segment.

Remember to read down the first column before reading the second column.

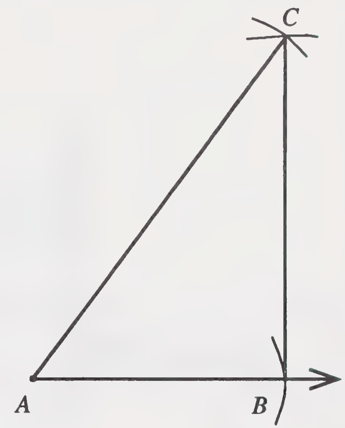
Step 6: Set your compass so that the point and the pencil are on the endpoints of s_3.



Step 7: Keeping the same setting, place the compass point at B and draw an arc to intersect the arc of Step 5. Label the intersection as point C.



Step 8: Join A to C and B to C to complete the triangle.

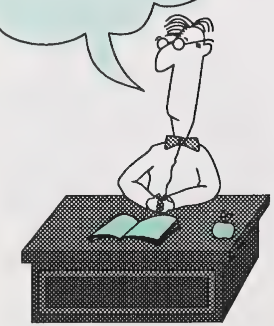


This construction is called construction by SSS because the three sides were given. Remember that SSS stands for Side Side Side.

The next example shows another construction by SSS.

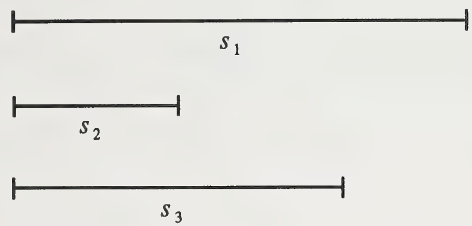
Your construction should show all the arcs that are needed to complete the task.

no arcs = no marks



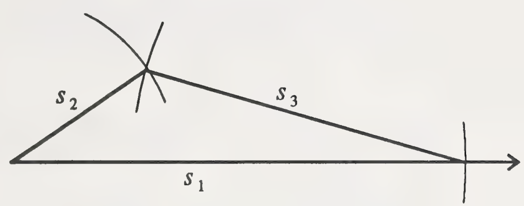
Example 1

Construct the triangle having sides congruent to the line segments shown.



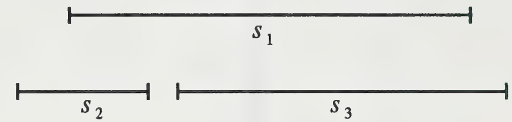
Solution:

Work through this construction step by step. Refer to the steps given in the previous construction if necessary.



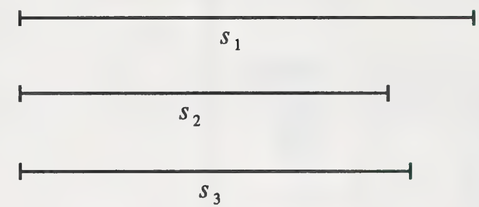
Now you will have a chance to try some constructions by SSS. Do at least the first two questions.

1. Construct a triangle from the line segments shown.



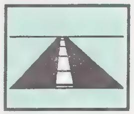
2. Construct a triangle from sides having lengths of 3 cm, 6 cm, and 7 cm.

3. Construct a triangle using the following line segments.



For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.

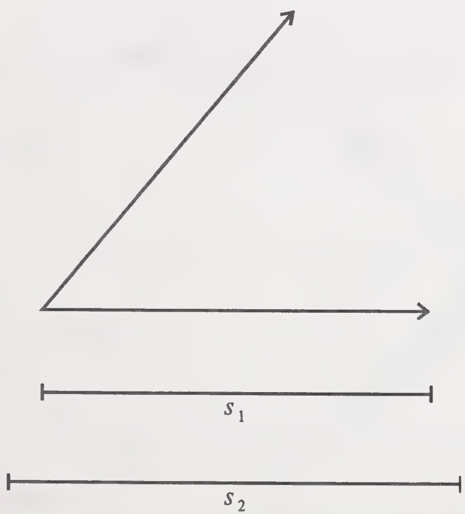
Activity 2



Construct a triangle given two sides and the included angle.

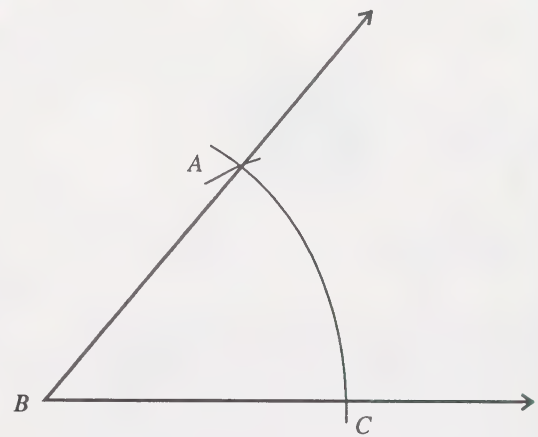
If you are given two sides and the included angle for a triangle, you can construct the triangle.

Construct a triangle having the angle and sides shown. The angle is formed by the two given sides. This is why it is called the included angle.



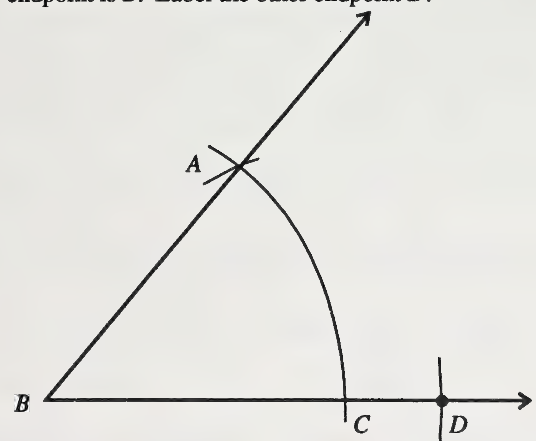
The steps for construction are as follows.

Step 1: Construct an angle congruent to the given angle. Make the sides longer than either of the given sides. Label the angle $\angle ABC$.

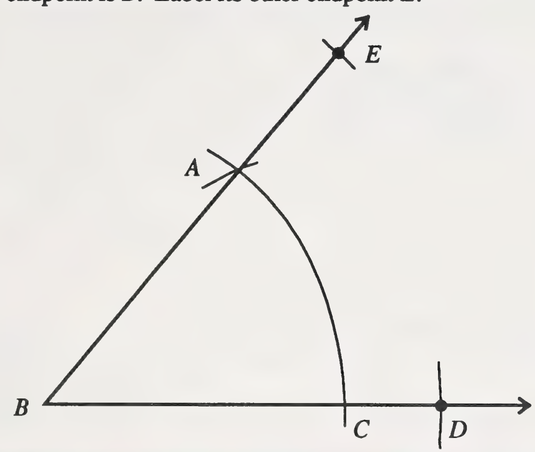


At this point you may wish to review the construction of congruent angles in Example 6 of the **What You Already Know** section.

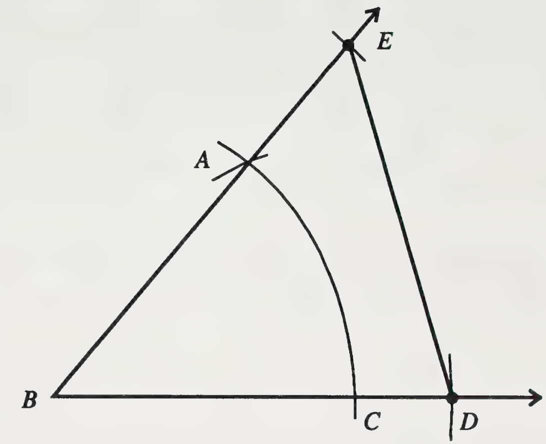
Step 2: On $\overrightarrow{BC}$ construct a line segment congruent with s_1. One endpoint is *B*. Label the other endpoint *D*.



Step 3: On $\overrightarrow{BA}$ construct a line segment congruent with s_2. One endpoint is *B*. Label its other endpoint *E*.



Step 4: Join points *E* and *D* to complete the triangle.

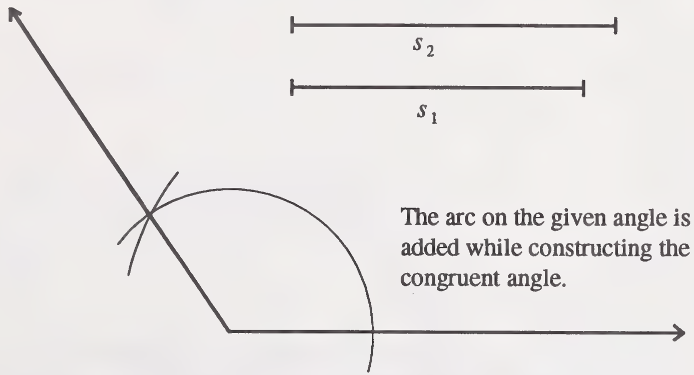


This construction is called construction by SAS because two sides and the included angle are given. Remember that SAS stands for Side Angle Side.

The following example shows how a construction by SAS is completed.

Example 2

Construct a triangle having sides congruent to the given line segments and having the included angle congruent to the given angle.



The arc on the given angle is added while constructing the congruent angle.

Solution:

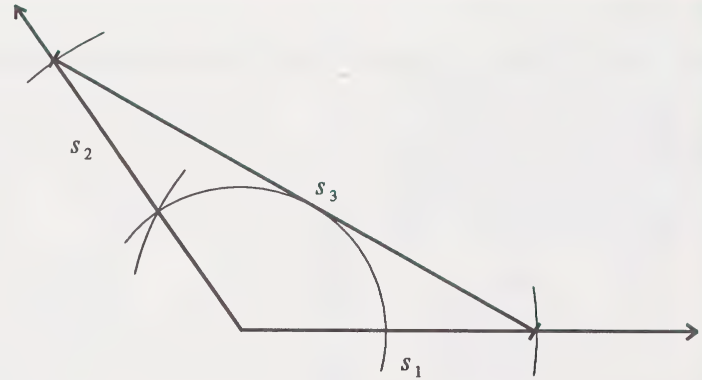
Follow through this construction step by step.

Step 1: Construct an angle congruent to the given angle.

Step 2: Construct a line segment congruent to the given side s_1.

Step 3: Construct a line segment congruent to the given side s_2.

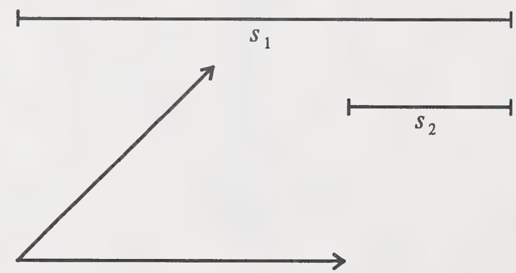
Step 4: Complete the triangle by making the third side or s_3 as shown.



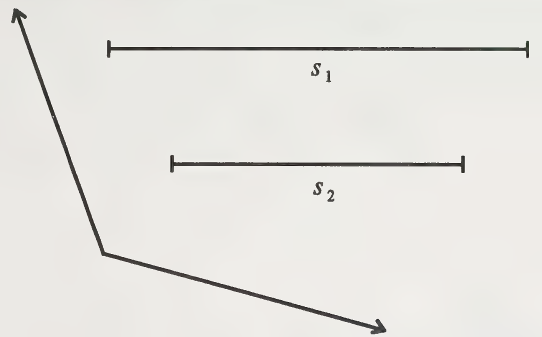
In the next questions you can apply construction by SAS on your own.

Do at least two of the following questions.

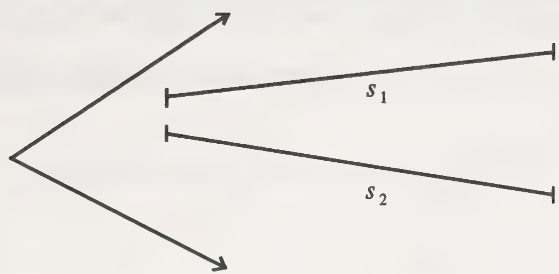
1. Construct a triangle having the line segments and the included angle given.



2. Use construction by SAS to construct a triangle having the line segments and the included angle shown.



3. Construct by SAS a triangle having the line segments and the included angle as follows.



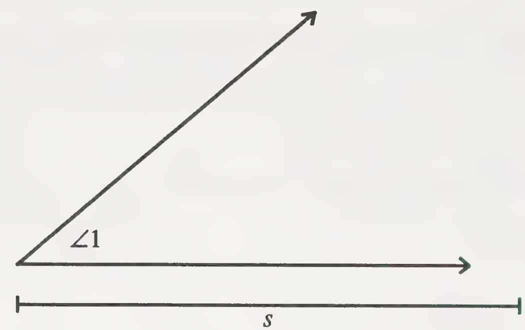
For solutions to **Activity 2**, turn to **Appendix A**, **Topic 4**.

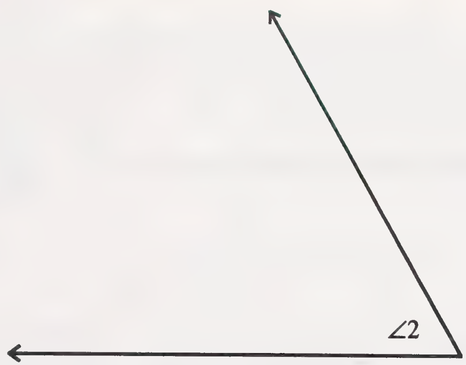
Activity 3

Construct a triangle given two angles and the included side.

Given two angles and the included side for a triangle, you can construct the triangle.

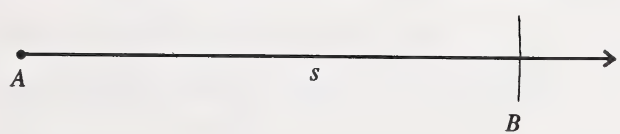
Construct a triangle having the angles and the included side as shown. The given side is to be between the vertices of the given angles.



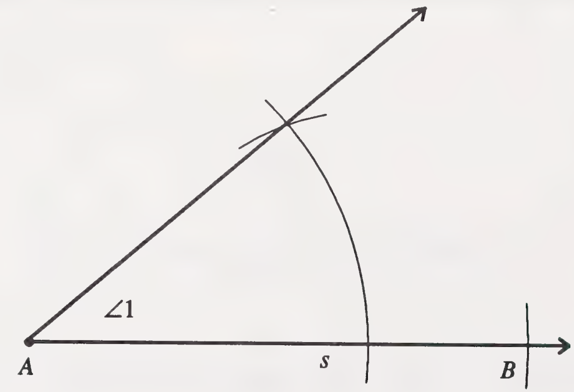


The steps for construction are as follows.

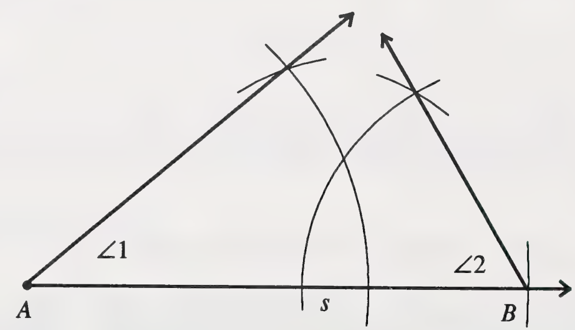
Step 1: Draw a ray that is longer than segment s. Construct a line segment congruent to s. Label its endpoints A and B.



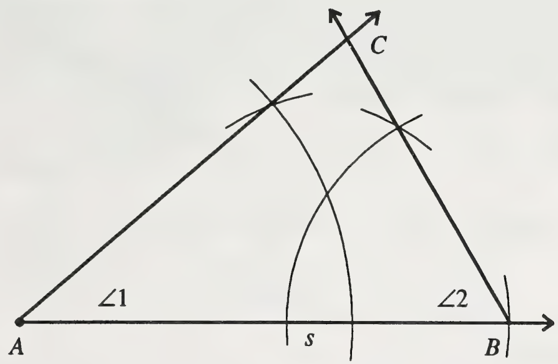
Step 2: Construct an angle with side AB and **vertex** A, congruent to $\angle 1$.



Step 3: Construct an angle with side AB and **vertex** B, congruent to $\angle 2$.



Step 4: Extend the rays of the two constructed angles until they intersect. Label the point of intersection C. The required triangle is ΔABC.

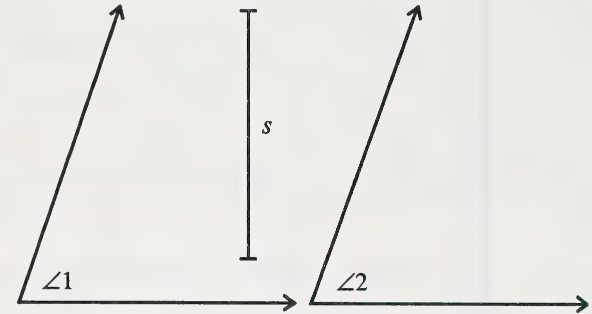


This is construction by ASA because two angles and the included side are given. Remember that ASA stands for Angle Side Angle.

The following example shows a triangle construction by ASA.

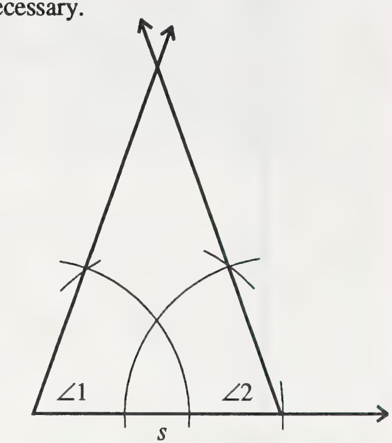
Example 3

Construct a triangle from the two angles and the included side shown.



Solution:

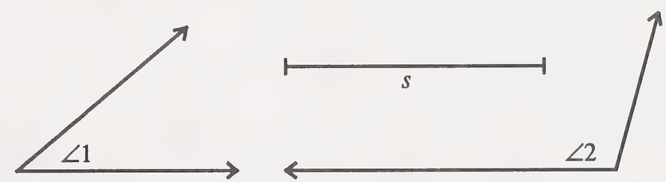
Follow through the construction steps. Refer to the steps in the previous construction if necessary.



Now do the following questions so you can apply construction by ASA.

Do any three of the four questions.

1. Construct a triangle having the given angles and the included side shown.



2. Suppose one given angle has a measure of 16°, the other given angle has a measure of 23°, and the given side has a length of 7.5 cm. If the given side is to be the included side, construct the triangle using the given side and angles.

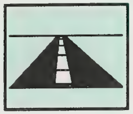
3. a. Draw two angles measuring 30° and 45°. Draw a line segment having a length of 10 cm. The given side is to be the included side.

   b. Construct a triangle using the parts you created in 3a so that the line segment is between the vertices of the angles.

4. Construct a triangle such that the two given angles measure 55° each. The included side is to be equal to a line segment measuring 6 cm.

For solutions to **Activity 3**, turn to **Appendix A, Topic 4**.

Activity 4



Construct a triangle congruent to a given triangle.

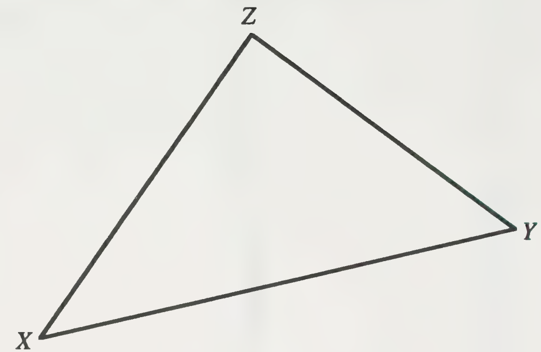
You have seen how triangles can be constructed given certain conditions. These conditions are as follows:

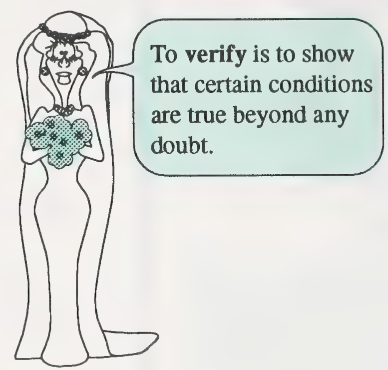
- Each of the sides is given (SSS).
- Two sides and the included angle are given (SAS).
- Two angles and the included side are given (ASA).

As you can see, you can talk about construction of triangles by SSS, by SAS, and by ASA. Notice that the abbreviations are the same as for the congruency conditions discussed in the previous topic. There is a reason for this. The fact that any one of these three conditions determines a particular triangle verifies the related congruency condition.

You can see how the conditions for triangle construction can be used to construct congruent triangles in this activity. You may find it helpful to use your compass to trace through each step in each construction.

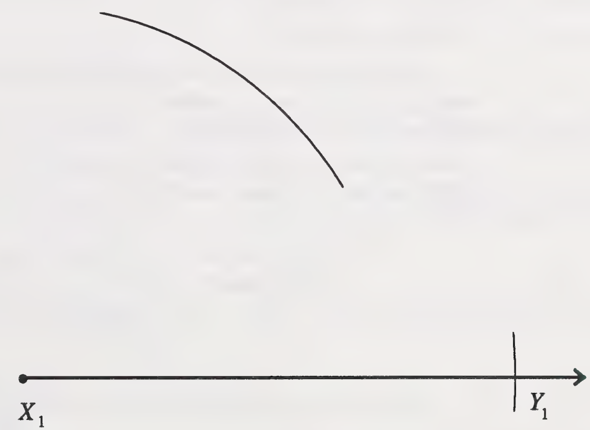
Construct a triangle congruent to ΔXYZ using the SSS property.

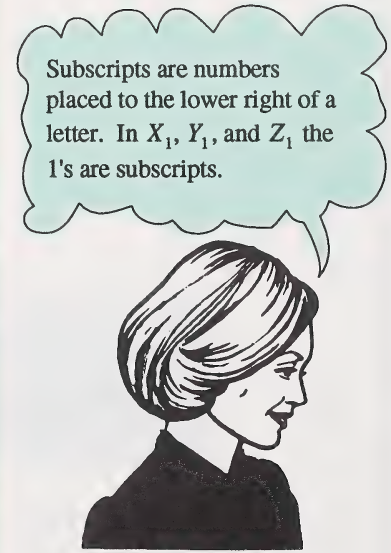




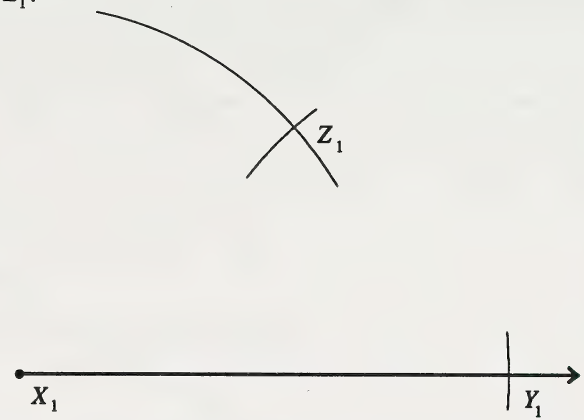
Step 1: Draw a ray longer than $\overline{XY}$. Mark a point X_1 on the left end. Construct a line segment congruent to $\overline{XY}$ by setting your compass so that the point and the pencil are on the endpoints of $\overline{XY}$ in the triangle. Using this compass setting, draw an arc on the line segment and label it Y_1.

Step 2: Set your compass so that the point and the pencil are on the endpoints of $\overline{XZ}$. Keeping the same setting, place the compass point at X_1 and draw an arc as shown.

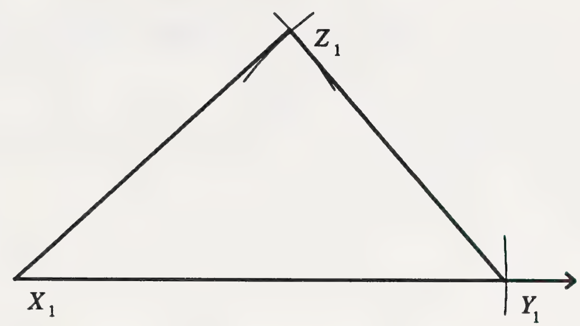




Step 3: Set the compass so that the point and the pencil are on the endpoints of $\overline{YZ}$. Keeping the same setting, place the compass point at Y_1 and draw an arc to intersect the previous arc. Label the intersection point Z_1.



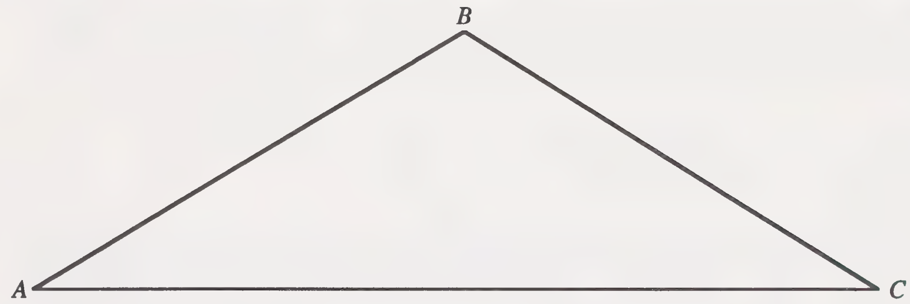
Step 4: Join X_1 to Z_1 and Y_1 to Z_1 to complete the triangle.



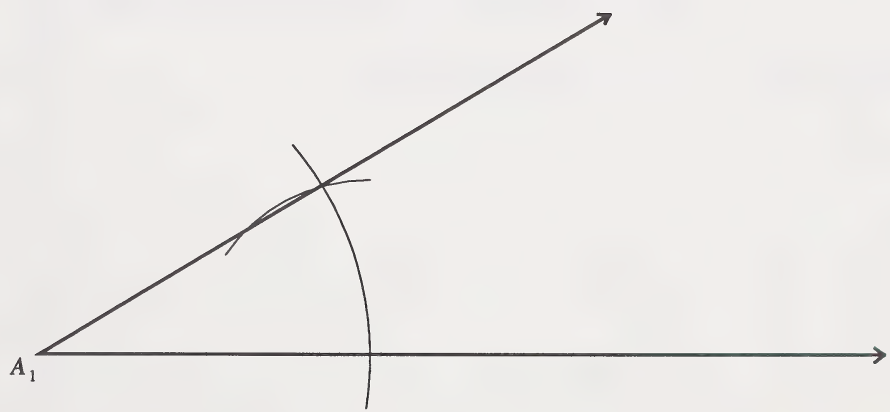
$\Delta XYZ \cong \Delta X_1 Y_1 Z_1$

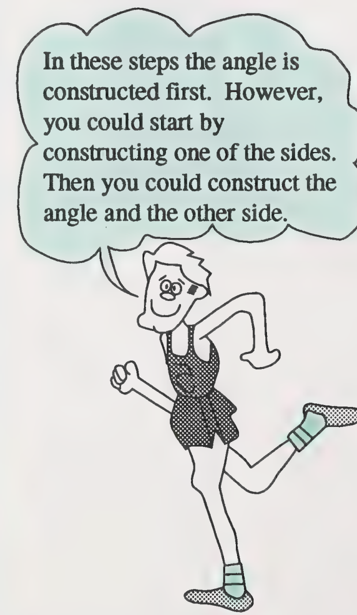
The new triangle is labelled using subscripts. The use of subscripts makes it easier to see the corresponding sides and angles.

Now see how to construct a triangle congruent to ΔABC using the SAS property. Use $\angle A$ as the included angle.

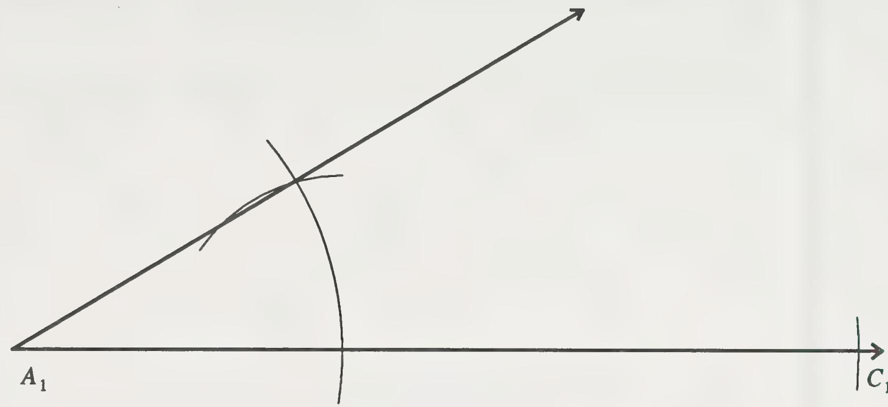


Step 1: Construct an angle congruent to $\angle A$. Label the vertex A_1.

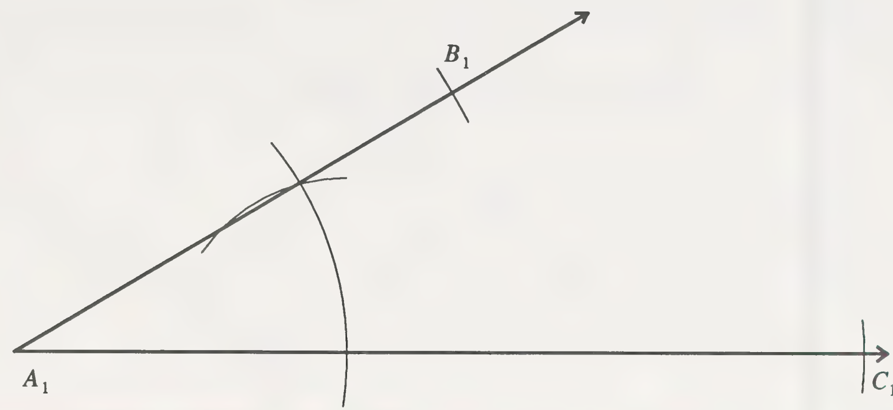




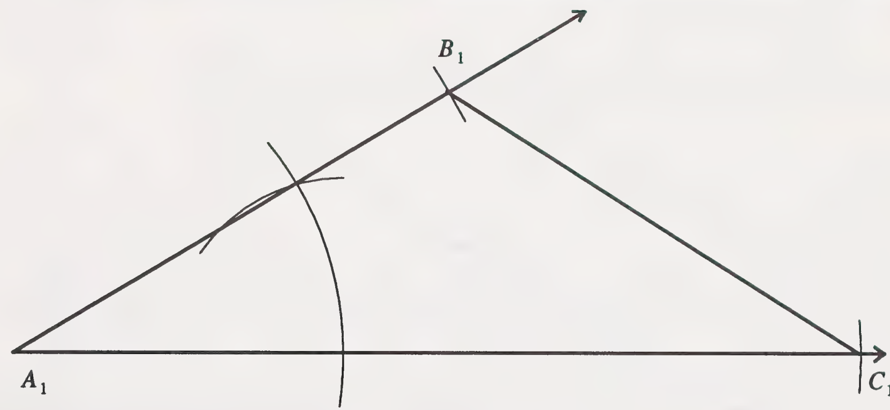
Step 2: On one of the angle sides construct a line segment congruent to $\overline{AC}$ with endpoint A_1. Label the other endpoint C_1.



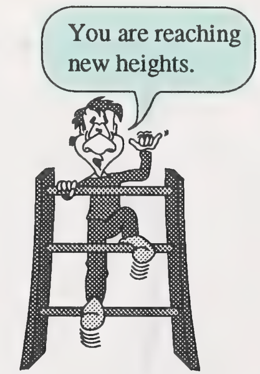
Step 3: On the other angle side construct a line segment congruent to $\overline{AB}$ with endpoint A_1. Label the other endpoint B_1.



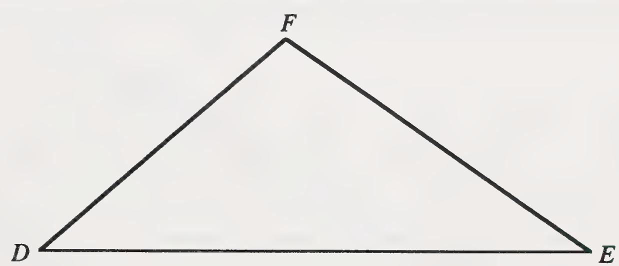
Step 4: Join B_1 and C_1 to complete the triangle.



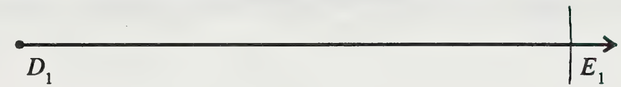
$\Delta ABC \cong \Delta A_1 B_1 C_1$



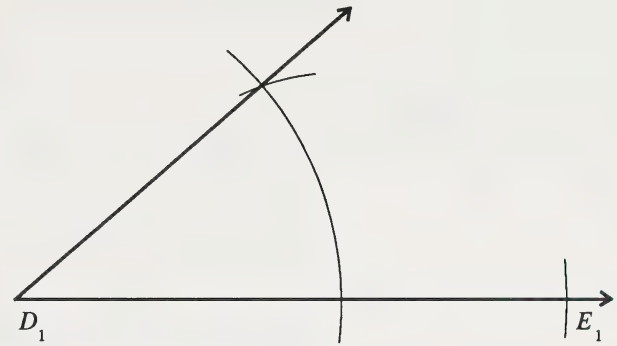
Work through the construction of a triangle congruent to ΔDEF using the ASA property. Use $\overline{DE}$ as the included side.



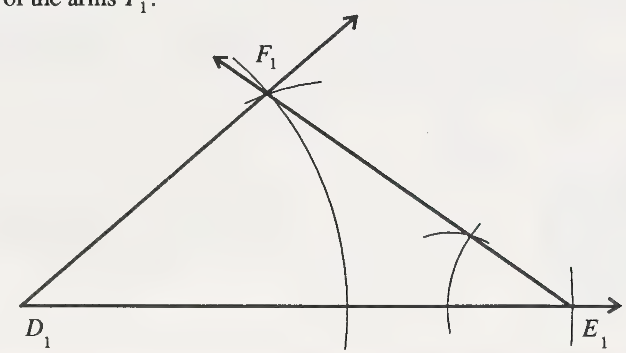
Step 1: Construct a line segment congruent to $\overline{DE}$ and label points D_1 and E_1 on it.



Step 2: Construct an angle from side $\overline{D_1E_1}$ that is congruent to $\angle D$ and has vertex D_1.



Step 3: Construct an angle from side $\overline{D_1E_1}$ that is congruent to $\angle E$ and has vertex E_1. Label the intersection of the arms F_1.



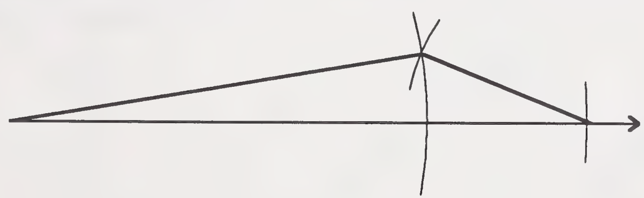
These constructions verify that two triangles are congruent when the congruence conditions hold for the triangles . The following examples show how this is applied.

Example 4

Construct a triangle congruent to the given triangle using the SSS property.

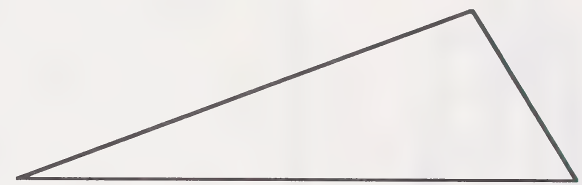
Solution:

Work through this construction step by step using a compass and a straightedge.



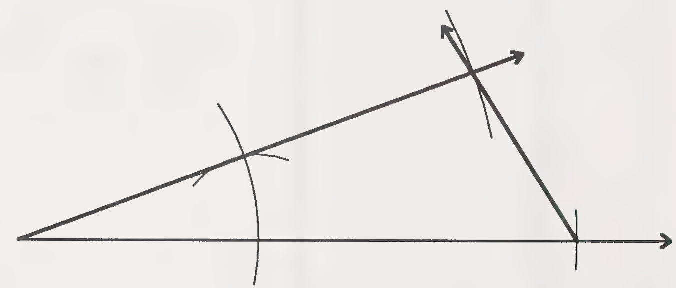
Example 5

Construct by SAS a triangle congruent to the given triangle.



Solution:

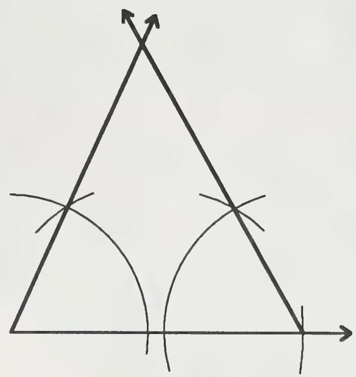
Work through the construction step by step using a compass and a straightedge.

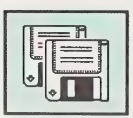


Example 6

Construct a triangle congruent to the given triangle using the ASA property.

Solution:





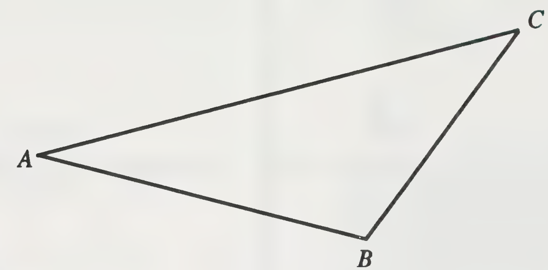
You may use the diskette entitled *Geometry – Congruent Triangles*,[1] Lesson 2.

[1] *Geometry – Congruent Triangles* is a title of Scott, Foresman & Company.

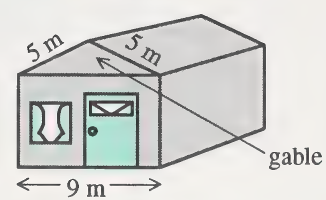
You can apply these constructions in the following questions.

Do at least the first four questions.

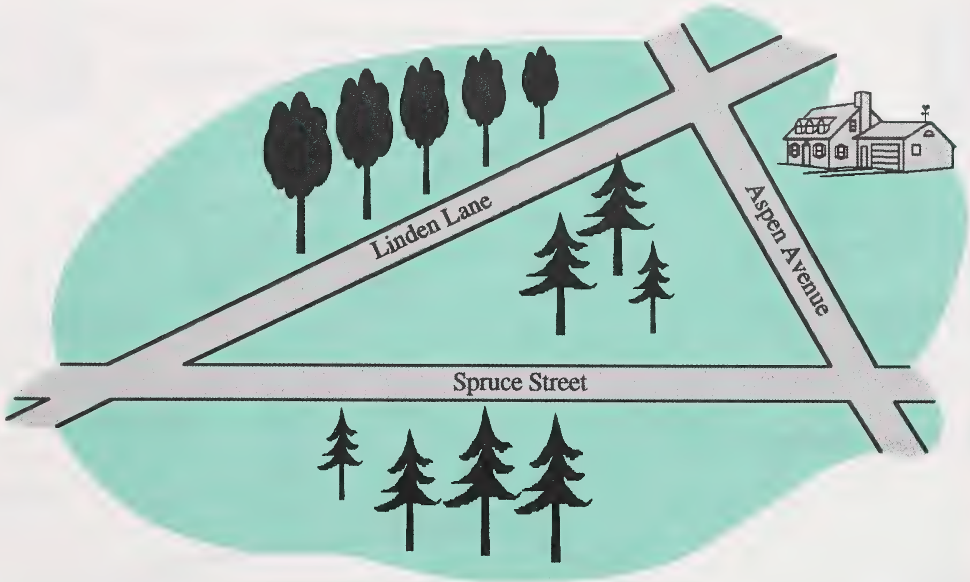
Use ΔABC for questions 1, 2, and 3.



1. Construct a triangle congruent to ΔABC by ASA.
2. Construct a triangle congruent to ΔABC by SSS.
3. Construct a triangle congruent to ΔABC by SAS.
4. Make a triangle to represent the gable of the building shown. Let 1 cm = 1 m for your scale drawing.

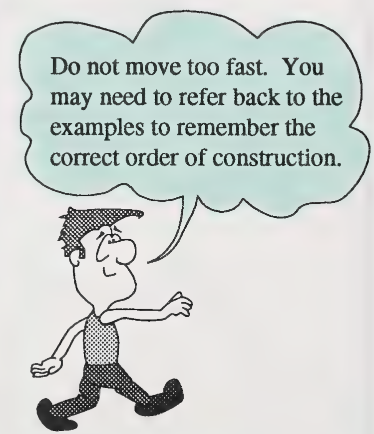


5. Linden Lane, Aspen Avenue, and Spruce Street cross to form a triangle as shown in the illustration. The intersections on Spruce Street are 1 km apart. Linden Lane and Spruce Street meet to form an angle of 25°. Spruce Street and Aspen Avenue form a 60° angle. Construct by ASA a triangle to represent the network in the picture. Let 1 cm = 100 m in your scale drawing.



6. Draw a triangle with two angles measuring 30° and 80°. The side between the vertices of these angles is to be 8 cm long. Construct by ASA a triangle congruent to the first triangle.

For solutions to **Activity 4**, turn to **Appendix A**, **Topic 4**.



If you require help, do the Extra Help section.

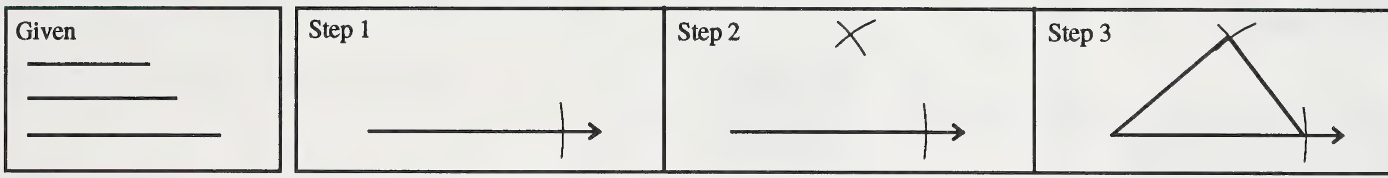
If you want more challenging explorations, do the Extensions section.

You may decide to do both.

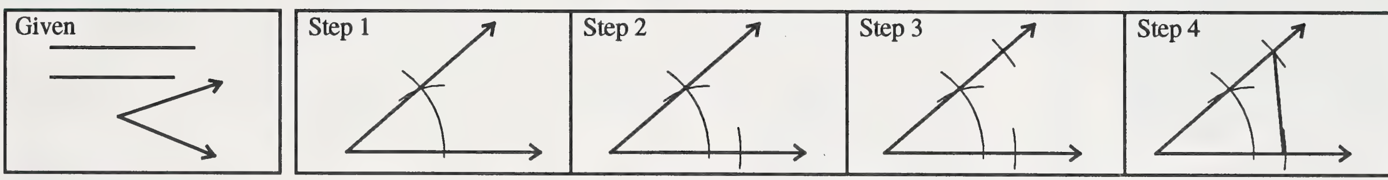
Extra Help

You may find it helpful to quickly scan the process of triangle construction. The SAS, ASA, and SSS constructions are shown in rapid succession here. There is no explanation provided. Each step simply shows you what is done.

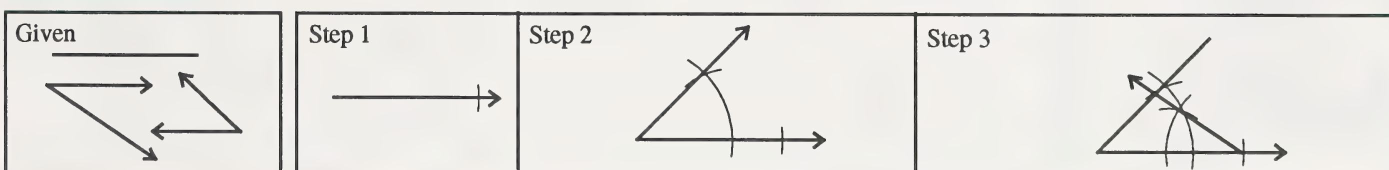
SSS Construction



SAS Construction

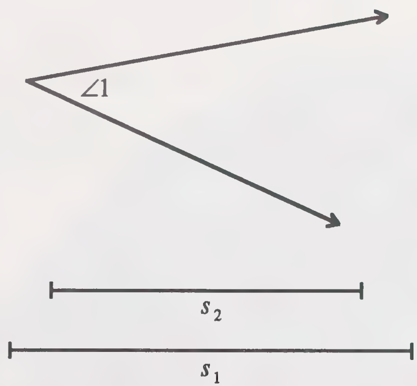


ASA Construction

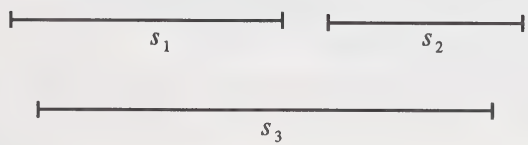


Using the construction chart as a guide, you can now construct triangles on your own in the following questions.

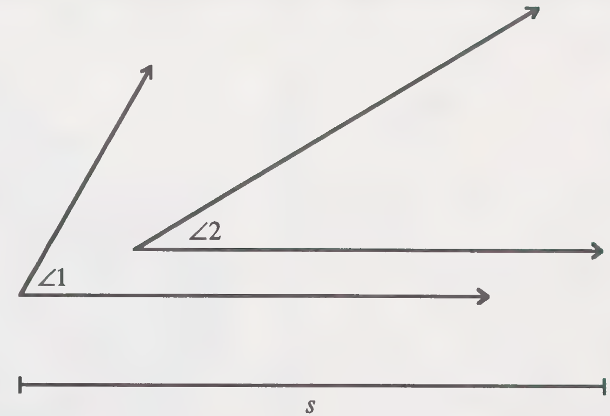
1. Construct a triangle given these parts.



2. Construct a triangle given these segments.



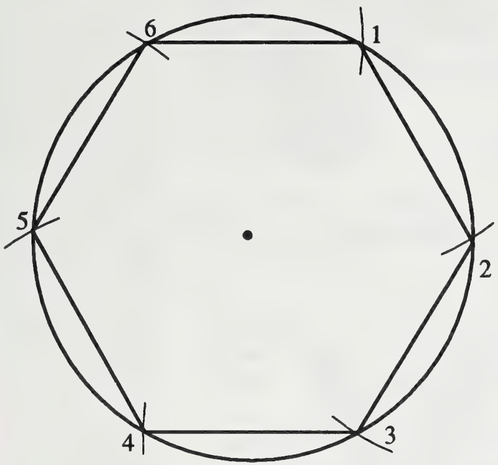
3. Construct a triangle given two angles and a segment.



For solutions to **Extra Help**, turn to **Appendix A, Topic 4**.

Extensions

You can make several interesting figures with a compass and a straightedge.



The **hexagon** can be made as follows.

Step 1: Draw a circle.

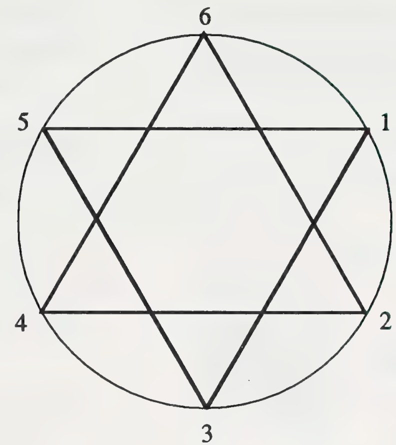
Step 2: Keeping the same compass setting, place the compass point at any point on the circle and intersect the circle with an arc at 1. In the preceding diagram the compass was placed at point 6 when arc 1 was made.

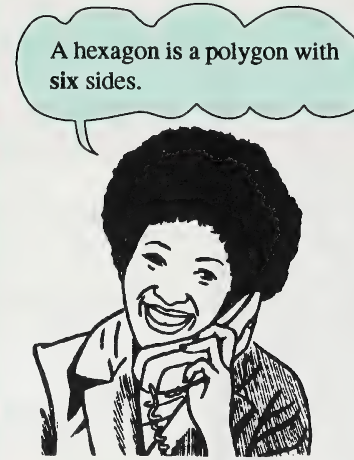
Step 3: Place the compass point at 1 and draw an arc to intersect the circle at 2.

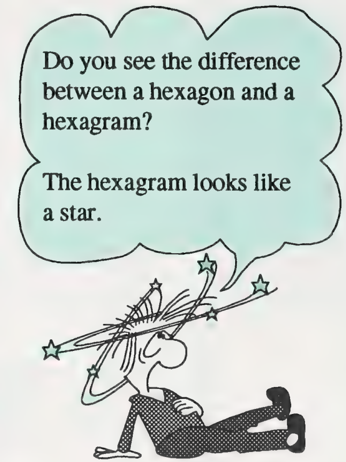
Step 4: Continue in this way until you have arcs intersecting the circle at 3, 4, 5, and 6.

Step 5: Join points 1 and 2, 2 and 3, and so on.

The next figure can be made by joining the points in a different way. See if you can construct the **hexagram** yourself.







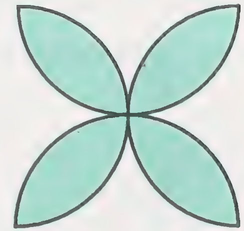
Now you will have the chance to make other figures. You may have to erase some lines to get the final product.

Make the following figures using only your compass and your straightedge. You can make them any size you wish.

1.

2.

3.



For solutions to **Extensions**, turn to **Appendix A, Topic 4**.

Unit Summary

What You Have Learned

You have learned to do the following:

- Understand the relationship between pairs of vertically opposite angles and how to use this relationship to calculate the size of angles.
- Understand the relationship between the angles made by cutting parallel lines with a transversal and how this relationship can be applied.
- Define similar triangles by investigating the special relationships between angles, and identify the corresponding sides and the corresponding angles.
- Investigate and identify the special relationship between corresponding sides, and solve for an unknown side.
- Solve practical problems involving an unknown side in similar triangles.
- Define congruency as applied to triangles, and identify corresponding parts of congruent triangles.
- Identify the conditions for congruence of two triangles, and solve problems involving congruent triangles.
- Construct a triangle given three sides.
- Construct a triangle given two sides and the included angle.
- Construct a triangle given two angles and the included side.
- Construct a triangle congruent to a given triangle.

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Angle Relationships

Topic 2 Similar Triangles

Topic 3 Congruent Triangles

Topic 4 Constructing Triangles

Appendix B
Manipulatives

Angles for Topic 1, Extra Help

Triangles for Topic 3, Activity 1

Building Figures for Topic 3, Activity 2

Appendix A
Solutions

Review

1. a. $\angle ABC = 14°$ b. $\angle FGH = 125°$

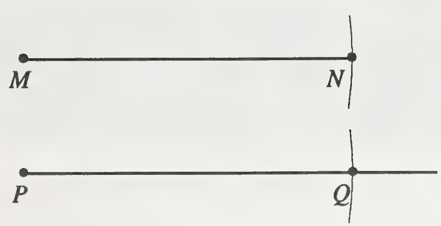
2. One pair of vertically opposite angles from the diagram is $\angle XYZ$ and $\angle VYW$. Another pair of vertically opposite angles is $\angle XYV$ and $\angle WYZ$.

3. $\angle B = 180° - (50° + 27°)$
   $= 180° - 77°$
   $= 103°$

4. a. $\angle A = 90° - 85°$
   $= 5°$

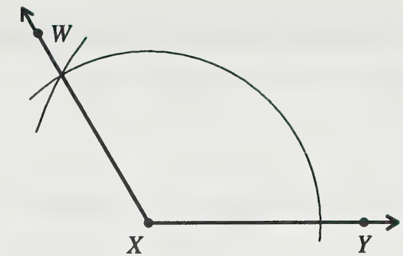
   b. $\angle E = 180° - 85°$
   $= 95°$

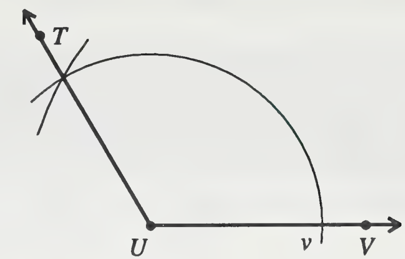
5.



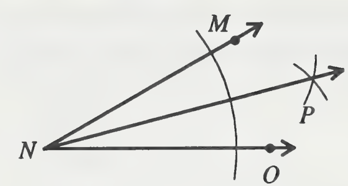
$\overline{MN} \cong \overline{PQ}$

6.





7.



Since $\angle MNP \cong \angle ONP$, $\overrightarrow{NP}$ is a bisector of $\angle MNO$.

Exploring Topic 1

Activity 1

Recognize the relationship between pairs of vertically opposite angles and how to use this relationsip to calculate the size of angles.

1. a. $\angle 1 = 104°$ and $\angle 3 = 104°$
   $\angle 2 = 76°$ and $\angle 4 = 76°$

   b. $\angle 5 = 164°$ and $\angle 7 = 164°$
   $\angle 6 = 16°$ and $\angle 8 = 16°$

   c. $\left.\begin{array}{l}\angle 9 = 90° \text{ and } \angle 11 = 90° \\ \angle 10 = 90° \text{ and } \angle 12 = 90°\end{array}\right\}$ These angles are **right angles**. A right angle has a measure of 90°.

   d. $\angle 13 = 40°$ and $\angle 15 = 40°$
   $\angle 14 = 140°$ and $\angle 16 = 140°$

2. Vertically opposite angles are equal in measure.

3. Since vertically opposite angles have the same measure, $\angle ACB = 35°$. Since the sum of the measures of the angles of any triangle is 180°, $\angle A$ is calculated as follows:

   $$\begin{aligned}\angle A &= 180° - (130° + 35°) \\ &= 15°\end{aligned}$$

4. Since vertically opposite angles have the same measure, $\angle DFE = 35°$ and $\angle DEF = 100°$. Since the sum of the measures of the angles of any triangle is 180°, $\angle D$ can be calculated as follows:

   $$\begin{aligned}\angle D &= 180° - (100° + 35°) \\ &= 45°\end{aligned}$$

5. Since $\angle 1$, $\angle 2$, and $\angle 3$ put together form a straight angle which measures 180°, $\angle 1$ can be calculated as follows:

   $$\begin{aligned}\angle 1 &= 180° - (90° + 70°) \\ &= 20°\end{aligned}$$

   Since vertically opposite angles are equal in measure, $\angle 4 = \angle 1 = 20°$.

Activity 2

Recognize the relationship among the angles made by cutting parallel lines with a transversal and how this relationship can be applied.

1. $\angle 1 = 78°$ $\quad$ $\angle 5 = 78°$
   $\angle 2 = 102°$ $\quad$ $\angle 6 = 102°$
   $\angle 3 = 102°$ $\quad$ $\angle 7 = 102°$
   $\angle 4 = 78°$ $\quad$ $\angle 8 = 78°$

2. Corresponding angles are equal in measure.

$\angle 1 = \angle 5 = 78°$
$\angle 2 = \angle 6 = 102°$
$\angle 4 = \angle 8 = 78°$
$\angle 3 = \angle 7 = 102°$

3. Alternate interior angles are equal in measure.

$\angle 3 = 102°$ and $\angle 6 = 102°$
$\angle 4 = 78°$ and $\angle 5 = 78°$

4. Same-side interior angles are supplementary.

Since $\angle 3 = 102°$ and $\angle 5 = 78°$, the sum of $\angle 3 + \angle 5 = 180°$.

Since $\angle 4 = 78°$ and $\angle 6 = 102°$, the sum of $\angle 4 + \angle 6 = 180°$.

5. Here your answers will be unique. Your angle measurements should support the relationships found in the previous four questions. The relationships are as follows:

- Corresponding angles are equal in measure.
- Alternate angles are equal in measure.
- Same-side interior angles are supplementary.

6. Since corresponding angles have the same measure, $\angle 1 = 59°$.

7. Since alternate interior angles are equal in measure, $\angle ABC = 40°$.

8. Since alternate interior angles are equal in measure, $x = 107°$.

9. a. Since corresponding angles are equal in measure, $\angle 2 = 78°$.

   b. Since vertically opposite angles are congruent, $\angle 1 = 78°$.

10. Since corresponding angles are equal in measure, $\angle ACB = \angle AED = 79°$.

The sum of the angles in any triangle is 180°.

$$\begin{aligned}\angle BAC &= 180° - (68° + 79°)\\ &= 180° - 147°\\ &= 33°\end{aligned}$$

11. Since alternate angles are equal in measure, $\angle CBD = \angle ADB = 33°$.

12. Since alternate angles are equal in measure, $\angle E = \angle A = 56°$.

Since the angles of a triangle add to 180°, $\angle DCE$ can be calculated as follows:

$$\begin{aligned}\angle DCE &= 180° - (56° + 80°)\\ &= 180° - 136°\\ &= 44°\end{aligned}$$

Since vertical angles are equal in measure,
$\angle ACB = \angle DCE = 44°$.

OR

Since alternate angles are equal in measure, $\angle B = \angle D = 80°$.

Since the sum of the angles in any triangle is 180°, $\angle ACB$ can be calculated as follows:

$$\begin{aligned}\angle ACB &= 180° - (56° + 80°)\\ &= 180° - 136°\\ &= 44°\end{aligned}$$

13. Yes, lines l and m are parallel since same-side interior angles are supplementary.

14. Since corresponding angles are equal in measure,
$\angle BEF = \angle CDE = 107°$.

Since the angles of a triangle add to 180°, $\angle EBF$ can be calculated as follows:

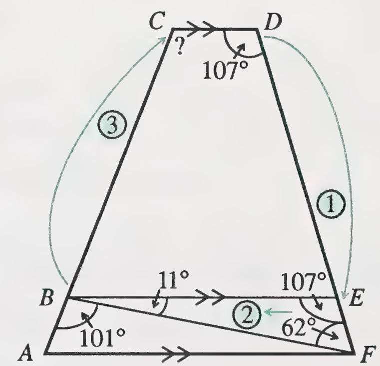
$$\begin{aligned}\angle EBF &= 180° - (107° + 62°)\\ &= 180° - 169°\\ &= 11°\end{aligned}$$

Since $\angle ABE$ is the sum of adjacent angles, it can be calculated as follows:

$$\begin{aligned}\angle ABE &= 101° + 11°\\ &= 112°\end{aligned}$$

Since corresponding angles are equal in measure,
$\angle BCD = \angle ABE = 112°$.

For an even better understanding follow the arrows in the next diagram.



15. The lines l and m are not parallel since the pair of corresponding angles are not equal in measure $(89° \neq 91°)$.

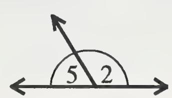
16. The angles marked with x's are alternate angles which are equal in measure. Therefore, to the sailor, the lighthouse keeper would appear to be 6° above the horizon.

Note: You may have to read the solutions two or three times to really understand how the answers were determined.

Extra Help

1. Corresponding angles are equal in measure.

2. Same-side interior angles are supplementary.



3. Alternate interior angles are equal in measure.

4. Vertically opposite angles are equal in measure.

5. Remember to record the measures of the angles as you find them.

   Since vertically opposite angles are equal in measure, $\angle 3 = \angle 1 = 131°$.

   Since $\angle 1$ and $\angle 2$ are supplementary angles, $\angle 2 = 180° - 131° = 49°$.

   Since $\angle 4$ and $\angle 2$ are vertically opposite angles, $\angle 4 = \angle 2 = 49°$.

   Since $\angle 5$ and $\angle 1$ are corresponding angles, $\angle 5 = \angle 1 = 131°$.

   Since $\angle 8$ and $\angle 4$ are corresponding angles, $\angle 8 = \angle 4 = 90°$.

   OR

   Since $\angle 8$ and $\angle 5$ are supplementary angles, $\angle 8 = 180° - 131° = 49°$.

   Since $\angle 7$ and $\angle 3$ are corresponding angles, $\angle 7 = \angle 3 = 131°$.

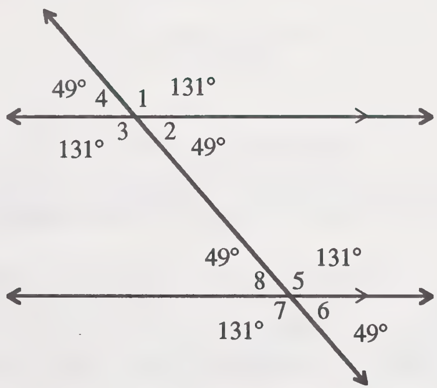
   OR

   Since $\angle 7$ and $\angle 8$ are supplementary angles, $\angle 7 = 180° - 49° = 131°$.

Since $\angle 6$ and $\angle 2$ are corresponding angles, $\angle 6 = \angle 2 = 49°$.

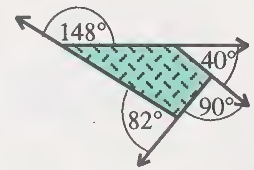
OR

Since $\angle 6$ and $\angle 7$ are supplementary angles,
$\angle 6 = 180° - 131° = 49°$.

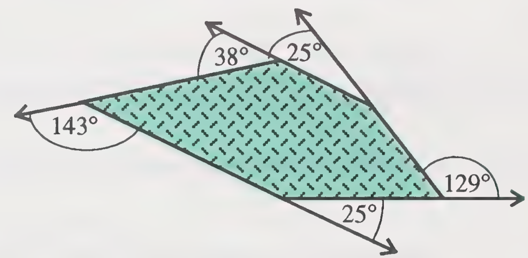


Extensions

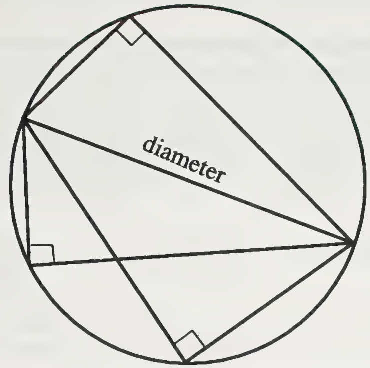
1. Your quadrilateral will not be the same as the one used here, but you should find that the sum of the measures of the exterior angles is 360°.



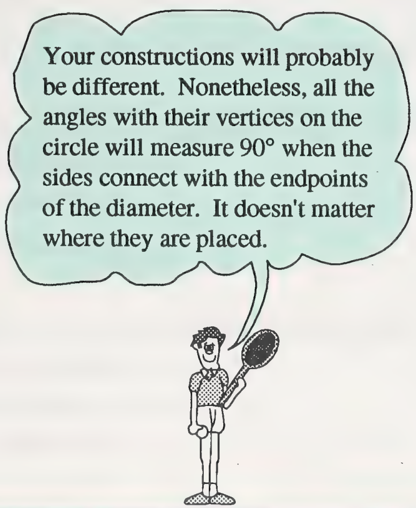
2. The sum should be 360° no matter what shape is used for the quadrilateral. For this five-sided polygon, or any other convex polygon, the sum of the measurements of the exterior angles always equals 360°.



3.



For any polygon in a circle, the angles formed at the vertices on the circle will have a measure of 90° when the sides connect with the endpoints of the diameter. Remember that the little square placed at a vertex indicates an angle which measures 90°.



Exploring Topic 2

Activity 1

Define similar triangles by investigating the special relationships between angles, and identify the corresponding sides and the corresponding angles.

1. $\angle A = 61°$ $\angle B = 34°$ $\angle C = 85°$

   $\angle D = 61°$ $\angle E = 34°$ $\angle F = 85°$

   Do not be concerned if your angle measurement is off by 1°.

2. $\angle G = 69° = \angle J$

   $\angle H = 51° = \angle K$

   $\angle I = 60° = \angle L$

3. $\angle A = \angle M = 28°$

   $\angle B = \angle N = 130°$

   $\angle C = \angle P = 22°$

   Since the corresponding angles in ΔABC and ΔNPM are equal in measure, $\Delta ABC \sim \Delta NPM$.

4. The corresponding angles and sides can be identified from the congruency statement.

   a. $\angle A$ corresponds to $\angle F$.
      $\angle B$ corresponds to $\angle D$.
      $\angle C$ corresponds to $\angle E$.
      In order to be corresponding, the angles must be congruent.

   b. $\overline{AB}$ corresponds to $\overline{FD}$.
      $\overline{BC}$ corresponds to $\overline{DE}$.
      $\overline{AC}$ corresponds to $\overline{FE}$.
      The corresponding sides are opposite corresponding angles.

5. For ΔABC and ΔADE, the angle relationships are as follows:

   Since $\angle ABC$ and $\angle ADE$ are corresponding angles resulting from a transversal intersecting two parallel lines, $\angle ABC = \angle ADE$.

   Since $\angle ACB$ and $\angle AED$ are corresponding angles resulting from a transversal intersecting two parallel lines, $\angle ACB = \angle AED$.

   Since $\angle BAC$ and $\angle DAE$ are the same angle, $\angle BAC = \angle DAE$.

   Since the corresponding angles in ΔABC and ΔADE are equal in measure, $\Delta ABC \sim \Delta ADE$.

6. a. Since alternate interior angles are equal in measure, $\angle N = \angle R$.

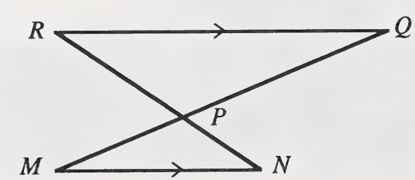
   Since alternate interior angles are equal in measure, $\angle M = \angle Q$.

   Since vertically opposite angles have the same measure, $\angle MPN = \angle QPR$.

   Since the corresponding angles in ΔPMN and ΔPQR are equal in measure, $\Delta PMN \sim \Delta PQR$.

   b. From the congruency statement, the corresponding sides can be identified.

   $\overline{PM}$ corresponds to $\overline{PQ}$.
   $\overline{MN}$ corresponds to $\overline{QR}$.
   $\overline{PN}$ corresponds to $\overline{PR}$.
   (Be careful to note that $\overline{PM}$ does not correspond to $\overline{PR}$.)



Activity 2

Investigate and identify the special relationship between corresponding sides, and solve for an unknown side.

1. $AB = 3.0$ cm, $BC = 1.5$ cm, $CA = 2.0$ cm

   $DE = 6.0$ cm, $EF = 3.0$ cm, $FD = 4.0$ cm

   $\frac{DE}{AB} = \frac{6.0\text{ cm}}{3.0\text{ cm}} = 2.0$ $\qquad \frac{EF}{BC} = \frac{3.0\text{ cm}}{1.5\text{ cm}} = 2.0$ $\qquad \frac{FD}{CA} = \frac{4.0\text{ cm}}{2.0\text{ cm}} = 2.0$

2. $NP = 4.0$ cm, $PQ = 2.0$ cm, $NQ = 5.0$ cm

   $RS = 10.0$ cm, $ST = 5.0$ cm, $ST = 12.5$ cm

   $\frac{RS}{NP} = \frac{10.0\text{ cm}}{4.0\text{ cm}} = 2.5$ $\qquad \frac{ST}{PQ} = \frac{5.0\text{ cm}}{2.0\text{ cm}} = 2.5$ $\qquad \frac{ST}{PQ} = \frac{12.5\text{ cm}}{5.0\text{ cm}} = 2.5$

3. For similar triangles, dividing corresponding side lengths results in the same quotients.

4. Since corresponding sides form the same ratio, $\frac{NP}{RS} = \frac{MN}{QR}$.

   Let $QR = s$.

   $$\frac{10}{30} = \frac{5}{s}$$
   $$10s = 30 \times 5$$
   $$s = \frac{30 \times 5}{10} = \frac{150}{10} = 15$$

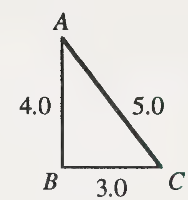
   $\therefore QR = 15$

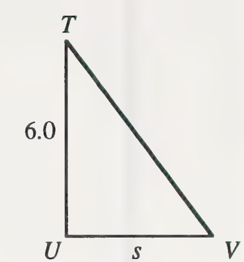
5. $\frac{AB}{TU} = \frac{BC}{UV}$

   Let $UV = s$.

   $$\frac{4.0}{6.0} = \frac{3.0}{s}$$
   $$4.0s = 6.0 \times 3.0$$
   $$s = \frac{6.0 \times 3.0}{4.0} = 4.5$$

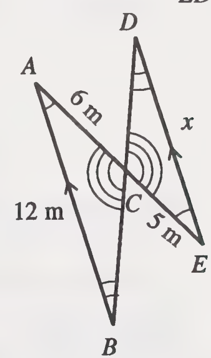
   $\therefore UV = 4.5$ units





6. Since $\angle A$ and $\angle E$ are alternate interior angles, $\angle A = \angle E$.
Since $\angle B$ and $\angle D$ are alternate interior angles, $\angle B = \angle D$.
Since $\angle ACB$ and $\angle DCE$ are vertically opposite angles, $\angle ACB = \angle DCE$.
Since the corresponding angles in ΔABC and ΔEDC are equal, $\Delta ABC \sim \Delta EDC$.

Since corresponding sides form the same ratio, $\frac{AB}{ED} = \frac{AC}{EC}$.



Let $DE = x$.

$$\frac{12}{x} = \frac{6}{5}$$
$$6x = 12 \times 5$$
$$x = \frac{12 \times 5}{6}$$
$$= 10$$

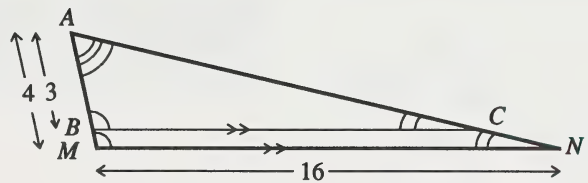
$\therefore DE = 10$ m

In ΔABC and ΔDEF, the side relationships are as follows:

- $\overline{AC}$ corresponds to $\overline{EC}$.
- $\overline{AB}$ corresponds to $\overline{ED}$.
- $\overline{BC}$ corresponds to $\overline{DC}$.

7. Since $\angle ABC$ and $\angle AMN$ are corresponding angles for parallel lines, $\angle ABC = \angle AMN$.
Since $\angle ACB$ and $\angle ANM$ are corresponding angles for parallel lines, $\angle ACB = \angle ANM$.
Since $\angle BAC$ and $\angle MAN$ are the same angle, $\angle BAC = \angle MAN$.
Since the corresponding angles in ΔABC and ΔAMN are equal, $\Delta ABC \sim \Delta AMN$.

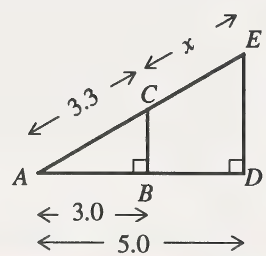
Let $BC = x$.



$$\frac{AB}{AM} = \frac{BC}{MN}$$
$$\frac{3}{4} = \frac{x}{16}$$
$$3 \times 16 = (4)x$$
$$x = \frac{3 \times 16}{4}$$
$$= 12$$

$\therefore BC = 12$ m

8. Since $\angle BAC$ and $\angle DAE$ are the same angle, $\angle BAC = \angle DAE$.
$\angle ABC = \angle ADE = 90°$ (given)
Since $\angle C$ and $\angle E$ are the third angles, $\angle C = \angle E$.
Since the corresponding angles of ΔABC and ΔADE are equal, $\Delta ABC \sim \Delta ADE$.



$$\frac{AB}{AD} = \frac{AC}{AE}$$
$$\frac{3.0}{5.0} = \frac{3.3}{x}$$
$$(3.0)x = 5.0 \times 3.3$$
$$x = \frac{5.0 \times 3.3}{3.0}$$
$$= 5.5$$

$\therefore AE = 5.5$ m

Since you know AE, you can find CE by subtracting AC from AE.

$$
\begin{aligned}
CE &= AE - AC \\
&= 5.5 \text{ m} - 3.3 \text{ m} \\
&= 2.2 \text{ m}
\end{aligned}
$$

Check:

$$\frac{AC}{AE} = \frac{AB}{AD}$$

$$\frac{3.3}{5.5} = \frac{3.0}{5.0}$$

$$3.3 \times 5.0 = 5.5 \times 3.0$$

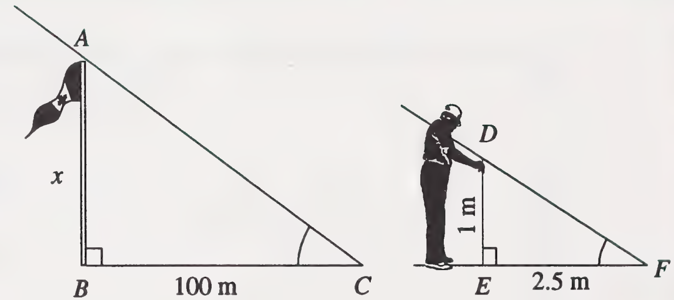
$$16.5 = 16.5$$

Therefore, $CE = 2.2$ m.

Activity 3

Solve practical problems involving an unknown side in similar triangles.

1.



$\angle B = \angle E = 90°$
Since light rays are parallel, $\angle C = \angle F$.
Since they are third angles, $\angle A = \angle D$.
It follows that $\Delta ABC \sim \Delta DEF$.

Let x be the height of the flagpole.
Since the corresponding sides form the same ratio, this proportion is used to do the calculation.

$$\frac{x}{1} = \frac{100}{2.5}$$

$$2.5x = 100$$

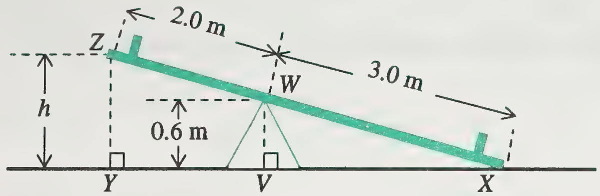
$$\frac{2.5x}{2.5} = \frac{100}{2.5} \times 1$$

$$x = 40$$

Therefore, the flagpole is 40 m high.

Notice that just as the shadow of the metre stick is longer than the metre stick itself, the shadow of the flagpole is longer than the flagpole itself.

2.



$\Delta XYZ \sim \Delta XVW$

$$\therefore \frac{XZ}{XW} = \frac{YZ}{VW}$$

$$\frac{2.0+3.0}{3.0} = \frac{h}{0.6}$$

$$3.0h = (2.0+3.0)\times 0.6$$

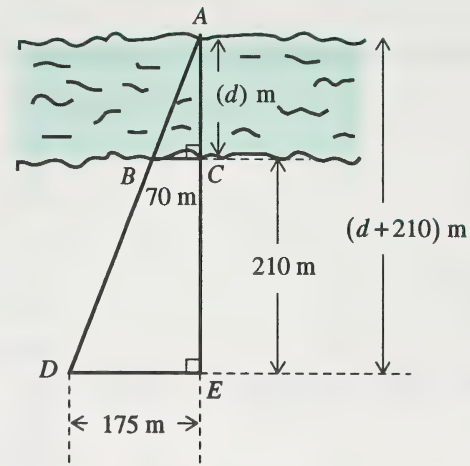
$$3.0h = 3.0$$

$$\frac{3.0h}{3.0} = \frac{3.0}{3.0}$$

$$h = 1$$

Therefore, the other end of the seesaw is 1 m off the ground.

3.



$\Delta ABC \sim \Delta ADE$

$$\therefore \frac{AC}{AE} = \frac{BC}{DE}$$

Since $AE = d + 210$, the proportion becomes as follows. Solve for d.

$$\frac{d}{d+120} = \frac{70}{175}$$

$$175d = 70(d+210) \quad \text{(from cross-multiplication)}$$

$$175d = 70d + 14\,700$$

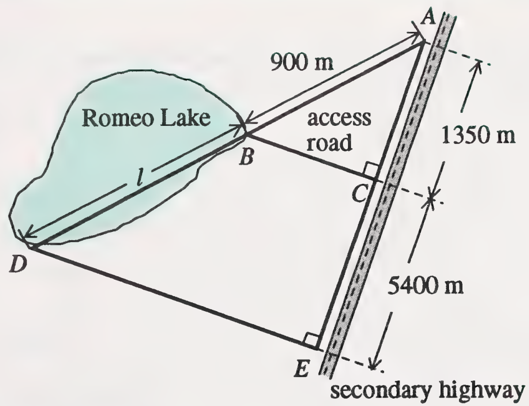
$$105d = 14\,700$$

$$d = \frac{14\,700}{105}$$

$$= 140$$

The width of the river is 140 m.

4.



$\Delta ADE \sim \Delta ABC$

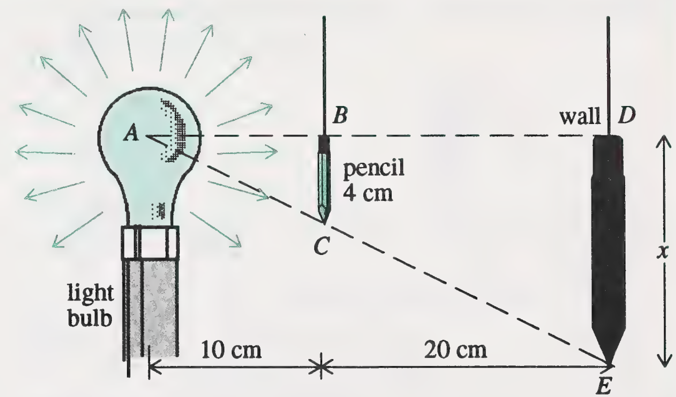
$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$

Since $AD = l + 900$ and $AE = 5400 + 1350$, the proportion becomes as follows. Solve for l.

$$\frac{l+900}{900} = \frac{5400+1350}{1350}$$
$$1350(l+900) = 900(5400+1350)$$
$$1350l + 1\,215\,000 = 900(6750)$$
$$1350l + 1\,215\,000 = 6\,075\,000$$
$$1350l = 6\,075\,000 - 1\,215\,000$$
$$1350l = 4\,860\,000$$
$$l = \frac{4\,860\,000}{1350}$$
$$= 3600$$

Romeo Lake has a length of 3600 m.

5. Let the shadow length be x.



Note that $\angle ABC$ and $\angle ADE$ are right angles since their sides are formed by a vertical line and a horizontal line.

Also $\angle A$ is common to ΔABC and ΔADE, so $\angle ACB$ and $\angle AED$ are third angles. It follows that $\Delta ABC \sim \Delta ADE$.

$\therefore \dfrac{AB}{AD} = \dfrac{BC}{DE}$

Since $AD = 10 + 20 = 30$, the proportion becomes as follows. Solve for x.

$$\frac{10}{30} = \frac{4}{x}$$
$$10x = 30 \times 4$$
$$x = \frac{30 \times 4}{10}$$
$$= 12$$

The length of the pencil's shadow is 12 cm.

What do you think would happen to the length of the shadow if the pencil was moved **closer** to the light source? If you are not sure, try this experiment for yourself.

Extra Help

1. $\dfrac{AB}{A^1B^1} = \dfrac{AC}{A^1C^1}$

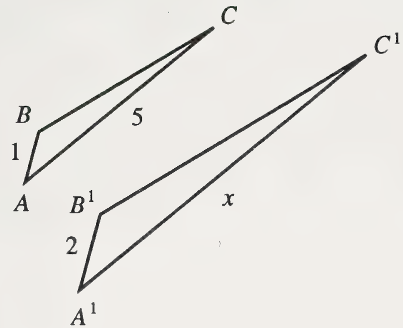
   Let $A^1C^1 = x$.

   $\dfrac{1}{2} = \dfrac{5}{x}$

   $x = 2 \times 5$

   $= 10$

   $\therefore A^1C^1 = 10$



2. $\Delta ABC \sim \Delta DEC$

   $\dfrac{AB}{DE} = \dfrac{AC}{DC}$

   Let $DC = x$.

   $\dfrac{10}{5} = \dfrac{8}{x}$

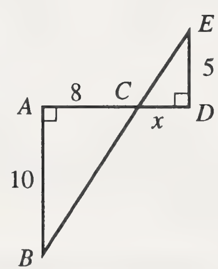
   $10x = 5 \times 8$

   $x = \dfrac{5 \times 8}{10}$

   $= \dfrac{40}{10}$

   $= 4$

   $\therefore DC = 4$



Notice that as AB is twice the length of DE; AC is also twice the length of DC.

3. $\dfrac{AB}{AB^1} = \dfrac{BC}{B^1C^1}$

   Let $AB^1 = x$.

   $\dfrac{22}{x} = \dfrac{14}{7}$

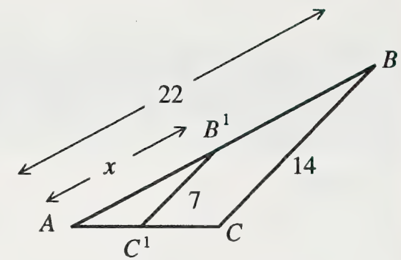
   $14x = 22 \times 7$

   $x = \dfrac{22 \times 7}{14}$

   $= \dfrac{154}{14}$

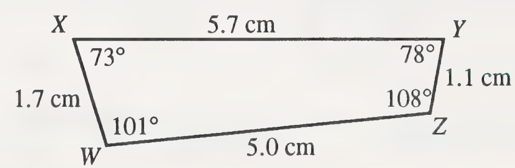
   $= 11$

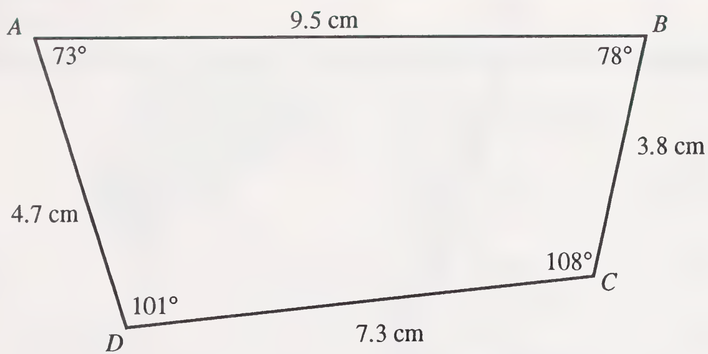
   $\therefore AB^1 = 11$



Extensions

1. $\angle X$ corresponds to $\angle A$. Both angles measure 73°.
   $\angle Y$ corresponds to $\angle B$. Both angles measure 78°.
   $\angle Z$ corresponds to $\angle C$. Both angles measure 108°.
   $\angle W$ corresponds to $\angle D$. Both angles measure 101°.





2. $\overline{XY}$ corresponds to $\overline{AB}$.

$$\therefore \frac{\overline{XY}}{\overline{AB}} = \frac{5.7}{9.5} \doteq 0.60$$

$\overline{YZ}$ corresponds to $\overline{BC}$.

$$\therefore \frac{\overline{YZ}}{\overline{BC}} = \frac{1.1}{3.8} \doteq 0.29$$

$\overline{WZ}$ corresponds to $\overline{DC}$.

$$\therefore \frac{\overline{WZ}}{\overline{DC}} = \frac{5.0}{7.3} \doteq 0.68$$

$\overline{WX}$ corresponds to $\overline{DA}$ and $\frac{\overline{WX}}{\overline{DA}} = \frac{1.7}{4.7} \doteq 0.36$.

Note that these are rounded to two decimal places.

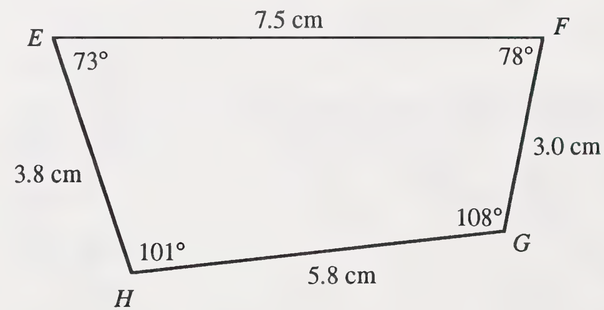
3. You can answer either with a yes or a no as to whether the two quadrilaterals are similar.

   Yes, they are similar because the corresponding angles have the same measure.

   OR

   No, they are not similar because the pairs of corresponding sides do not make the same ratio. However, in some cases two quadrilaterals will have congruent corresponding angles and pairs of corresponding sides that have the same ratio.

4. $\angle A$ corresponds to $\angle E$. Both angles measure 73°.
   $\angle B$ corresponds to $\angle F$. Both angles measure 78°.
   $\angle C$ corresponds to $\angle G$. Both angles measure 108°.
   $\angle D$ corresponds to $\angle H$. Both angles measure 101°.



5. $\overline{AB}$ corresponds to $\overline{EF}$.

$$\therefore \frac{\overline{AB}}{\overline{EF}} = \frac{9.5}{7.5} \doteq 1.27$$

$\overline{BC}$ corresponds to $\overline{FG}$.

$$\therefore \frac{\overline{BC}}{\overline{FG}} = \frac{3.8}{3.0} \doteq 1.27$$

$\overline{CD}$ corresponds to $\overline{GH}$.

$$\therefore \frac{\overline{CD}}{\overline{GH}} = \frac{7.3}{5.8} \doteq 1.26$$

$\overline{AD}$ corresponds to $\overline{EH}$.

$$\therefore \frac{\overline{AD}}{\overline{EH}} = \frac{4.7}{3.8} \doteq 1.24$$

These are rounded to two decimal places. Note the differences between the similarity requirements for triangles and quadrilaterals.

6. The quadrilaterals are similar since corresponding angles are equal in measure and pairs of corresponding sides form the same ratio. Actually the ratios are not **exactly** equal, but you must allow for error in measurement.

Exploring Topic 3

Activity 1

Define congruency as applied to triangles, and identify corresponding parts of congruent triangles.

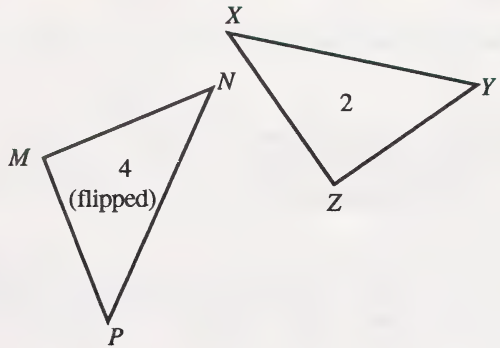
1. $\Delta 1$ fits over $\Delta 12$.
   $\Delta 3$ fits over $\Delta 5$.
   $\Delta 4$ fits over $\Delta 2$.
   $\Delta 6$ fits over $\Delta 8$.
   $\Delta 7$ fits over $\Delta 9$. (One of the triangles must be flipped.)
   $\Delta 13$ fits over $\Delta 14$. (One of the triangles must be flipped.)

2. Congruent triangles are same-size similar triangles.

3. Corresponding sides of congruent triangles have the same length.

4. a. $\angle X \leftrightarrow \angle N$ $\quad \overline{XY} \leftrightarrow \overline{NP}$
   $\angle Y \leftrightarrow \angle P$ $\quad \overline{YZ} \leftrightarrow \overline{PM}$
   $\angle Z \leftrightarrow \angle M$ $\quad \overline{XZ} \leftrightarrow \overline{NM}$

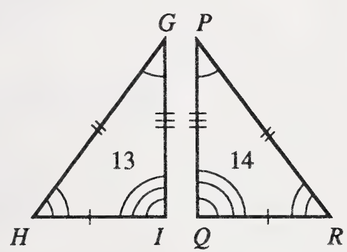
   b. Complete the congruency statement $\Delta MNP \cong$ $\underline{\Delta ZXY}$.



5. a. $\angle J \leftrightarrow \angle W$ $\quad \overline{JK} \leftrightarrow \overline{WV}$
$\angle K \leftrightarrow \angle V$ $\quad \overline{KL} \leftrightarrow \overline{VU}$
$\angle L \leftrightarrow \angle U$ $\quad \overline{JL} \leftrightarrow \overline{WU}$

b. Complete the congruency statement $\Delta JKL \cong$ **ΔWVU** .

6. a.



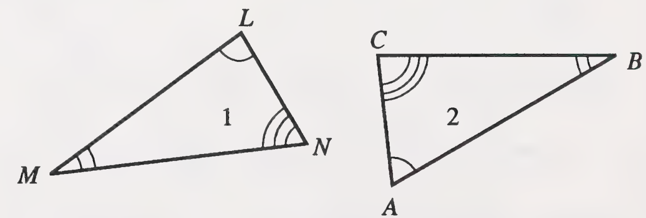
b. $\Delta GHI \cong$ **ΔPRQ**

The congruency statement could also be written as $\Delta HGI \cong \Delta RPQ$ or $\Delta IHG \cong \Delta QRP$.

7. From $\Delta LMN \cong \Delta ABC$,

$\angle L \leftrightarrow \angle A$
$\angle M \leftrightarrow \angle B$
$\angle N \leftrightarrow \angle C$

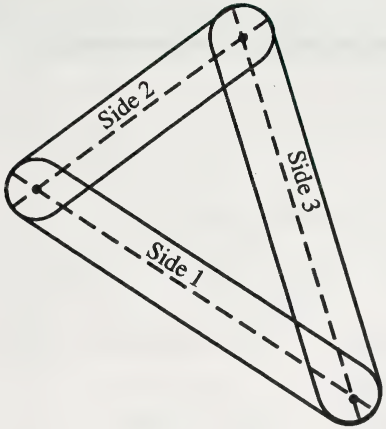
Therefore, the triangle must be labelled as follows:



Activity 2

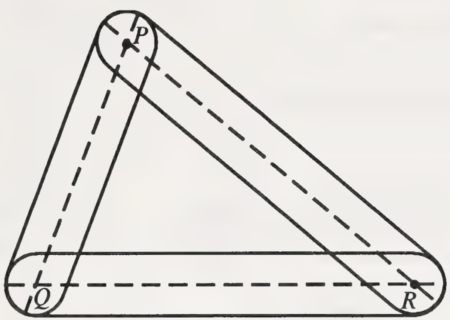
Identify the conditions for congruence of two triangles, and solve problems involving congruent triangles.

1. a.



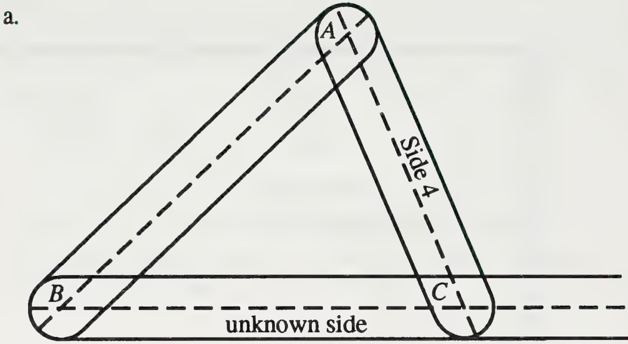
b. No, there is no different triangle that can be made.

2. a.

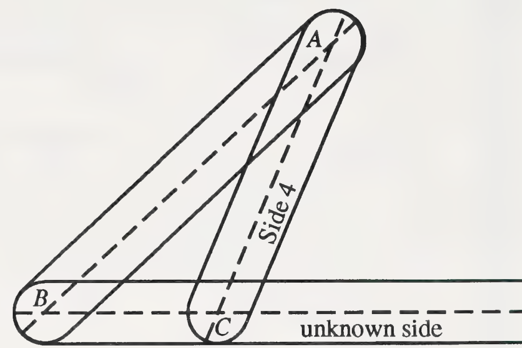


b. No, there is no different triangle that can be made.

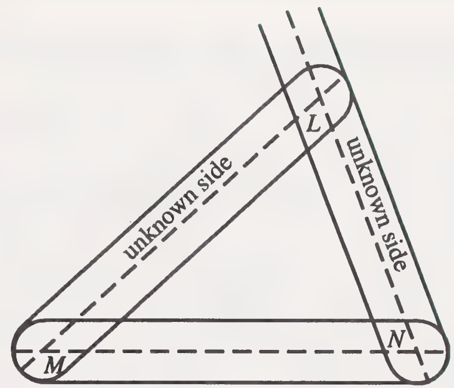
3. a.



b. Yes, there is one other triangle that can be built and it is shown here.



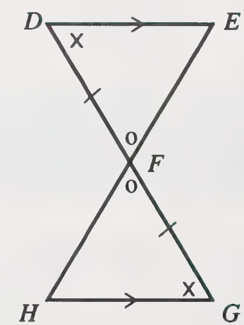
4. a.



b. No, there is no different triangle that can be built with these parts.

5.

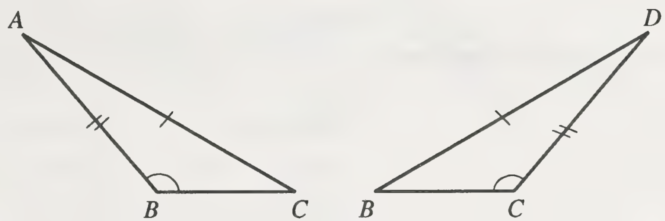
$\angle G = \angle D$	alternate angles for parallel line segments (A)
$FG = FD$	given (S)
$\angle GFH = \angle DFE$	vertically opposite angles (A)
$\Delta GHF \cong \Delta DEF$	(ASA)
$\therefore FH = FE$	corresponding sides of congruent triangles
and $FH = 2$	substituting 2 for FE



Alternate angles are labelled using x.

6. $\angle ABC = \angle DCB$ given (A)
$BC = BC$ common side (S)
$\angle ACB = \angle DBC$ given (A)
$\therefore \Delta ABC \cong \Delta DCB$ (ASA)

7. $BC = BC$ common side (S)
$\angle ABC = \angle DCB$ given (A)
$AB = DC$ given (S)
$\therefore \Delta ABC \cong \Delta DCB$ (SAS)



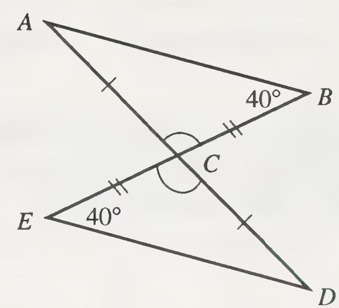
8. $\angle N = \angle Q$ given (A)
$MN = MQ$ given (S)
$\angle NMP = \angle QMR$ vertically opposite angles (A)
$\therefore \Delta MNP \cong \Delta MQR$ (ASA)

9. $AB = AD$ given (S)
$BC = DC$ given (S)
$AC = AC$ common side (S)
$\Delta ABC \cong \Delta ADC$ (SSS)

10. $PQ = MN$ given (S)
$\angle NQP = \angle QNM$ given (A)
$NQ = NQ$ common side (S)
$\therefore \Delta PQN \cong \Delta MNQ$ (SAS)

Therefore, $NP = QM$ since they are corresponding sides of congruent triangles. It follows that $NP = 4$ by substitution since $QM = 4$ is given.

11. $AC = DC$ given (S)
$\angle DCE = \angle ACB$ vertically opposite angles (A)
$BC = EC$ given (S)
$\therefore \Delta DEC \cong \Delta ABC$ (SAS)



Therefore, $\angle CED = \angle CBA$ since they are corresponding angles of congruent triangles. It follows that $\angle CED = 40°$ by substitution since $\angle CBA = 40°$ is given.

Extra Help

1. Triangle 1

$\angle G = 114°$	$GH = 4.2$ cm
$\angle H = 42°$	$HI = 9.5$ cm
$\angle I = 24°$	$GI = 7.1$ cm

Triangle 2

$\angle D = 28°$	$DE = 7.9$ cm
$\angle E = 116°$	$EF = 6.2$ cm
$\angle F = 36°$	$DF = 11.9$ cm

Triangle 3

$\angle A = 36°$	$AB = 11.9$ cm
$\angle B = 28°$	$BC = 7.9$ cm
$\angle C = 116°$	$AC = 6.2$ cm

2. Since for triangles 2 and 3 there is a correspondence for the sides and a correspondence for the angles, $\Delta 2 \cong \Delta 3$.

3.

$\angle A \leftrightarrow \angle F$	$\overline{AB} \leftrightarrow \overline{FD}$
$\angle B \leftrightarrow \angle D$	$\overline{BC} \leftrightarrow \overline{DE}$
$\angle C \leftrightarrow \angle E$	$\overline{AC} \leftrightarrow \overline{FE}$

4. $\Delta ABC \cong \Delta$ __FDE__
Corresponding angles have letters in corresponding positions.

Extensions

1.

$\angle ABC = \angle ADC$	given (right angles)
$AC = AC$	common side (H)
$AB = AD$	given (L)
$\therefore \Delta ABC \cong \Delta ADC$	(HL)

2.

$\angle EFG = \angle GHE$	given (right angles)
$EG = EG$	common side (H)
$EH = GF$	given (L)
$\therefore \Delta EFG \cong \Delta GHE$	(HL)

3.

$\angle XYZ = \angle MNP$	given (right angles)
$XZ = MP$	given (H)
$YZ = NP$	given (L)
$\therefore \Delta XYZ \cong \Delta MNP$	(HL)

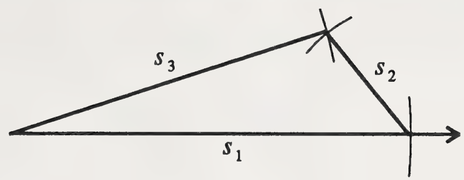
Since they are corresponding sides of congruent triangles, $XY = MN$.

Exploring Topic 4

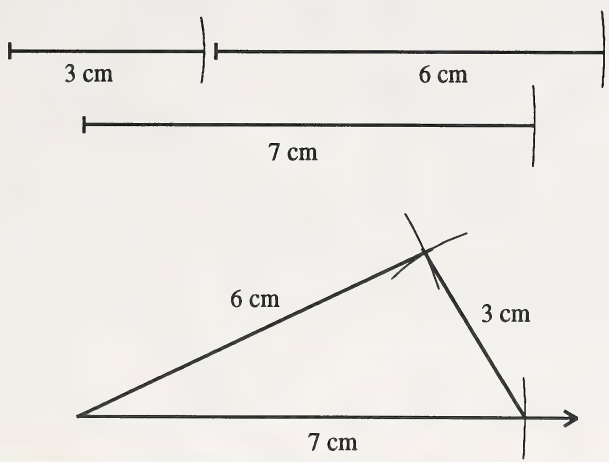
Activity 1

Construct a triangle given three sides.

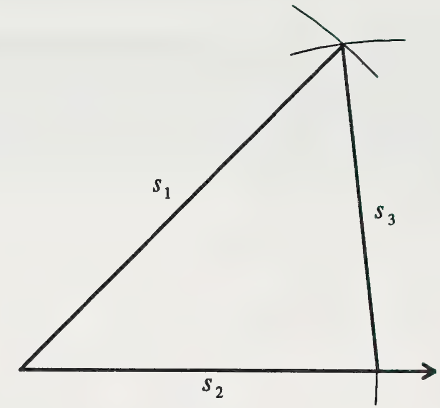
1.



2.



3.

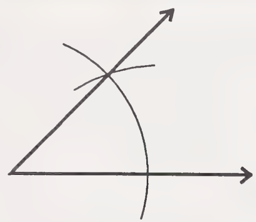


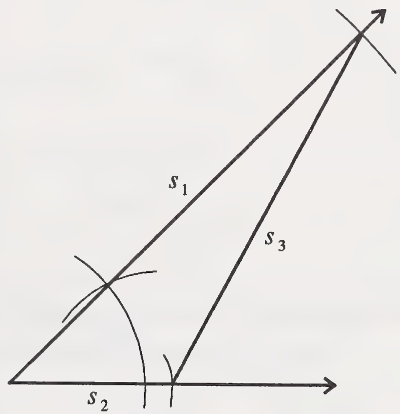
In this case, s_2 was constructed first. Any of the three sides could be constructed first. The resulting triangle is simply in a different position.

Activity 2

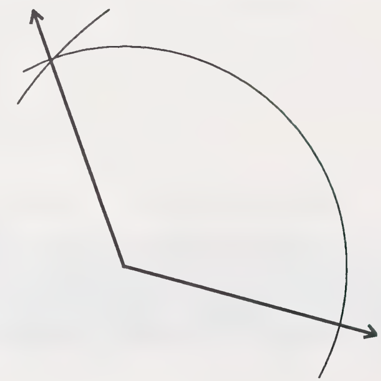
Construct a triangle given two sides and the included angle.

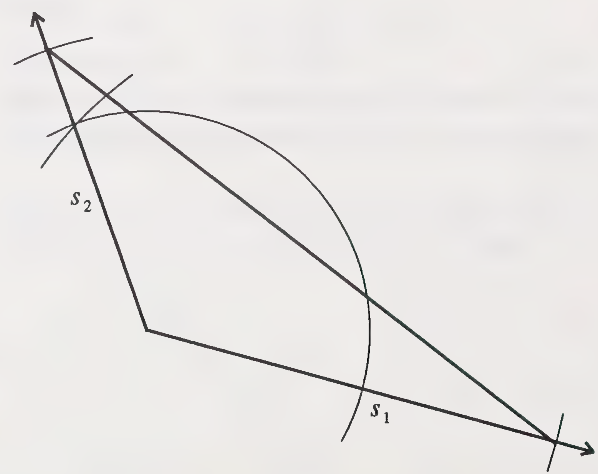
1. The congruent angle is constructed as follows:



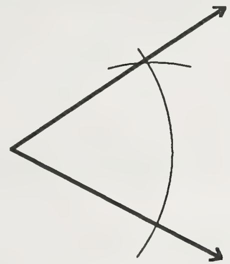


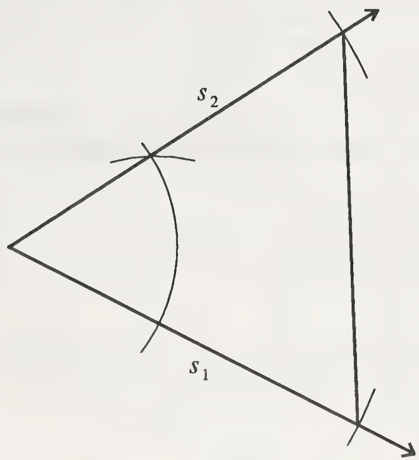
2. The congruent angle is constructed as follows:





3. The congruent angle is constructed as follows:

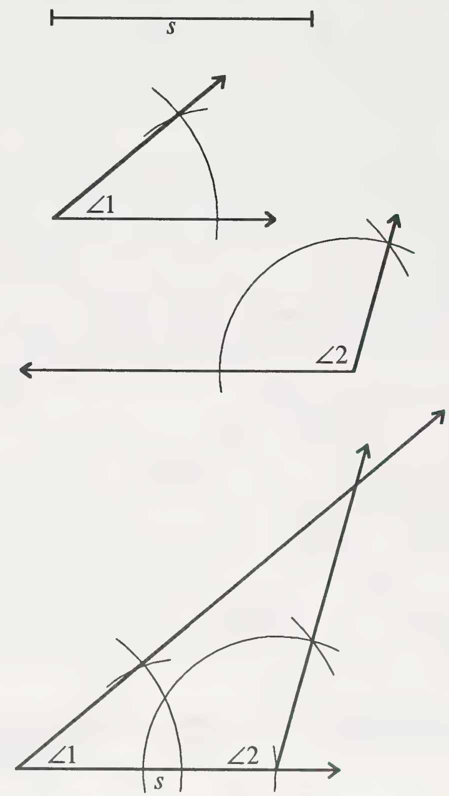




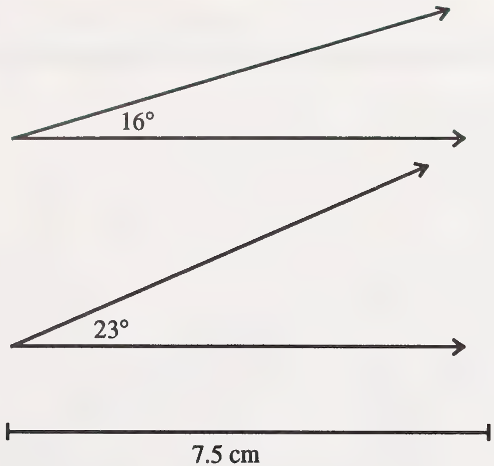
Activity 3

Construct a triangle given two angles and the included side.

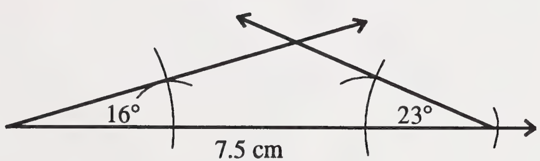
1.



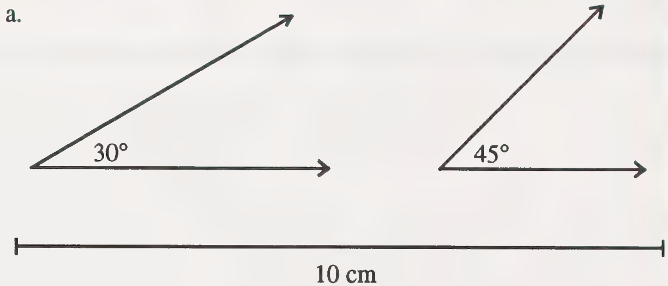
2.



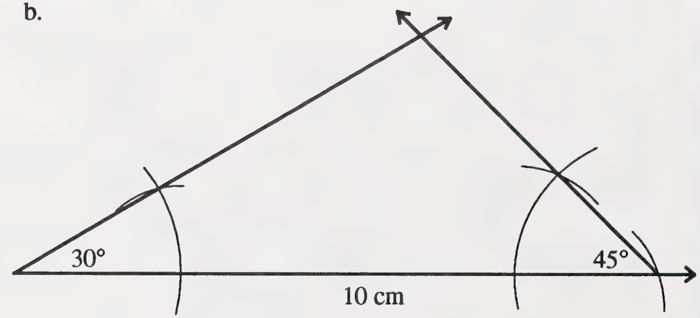
Remember to begin with the side construction. Then construct the two angles.



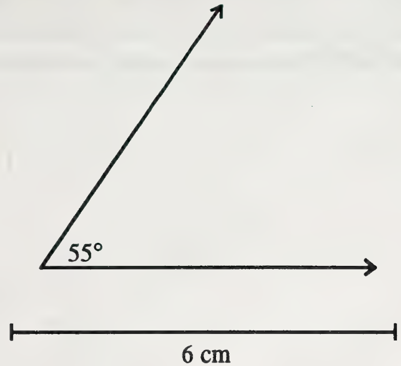
3. a.

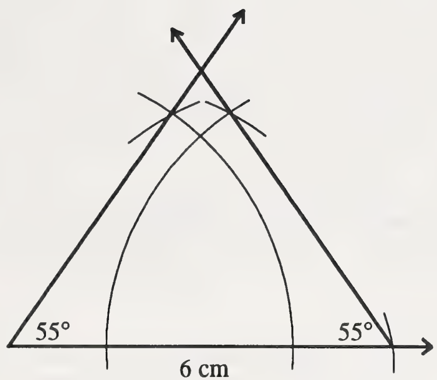


b.



4.

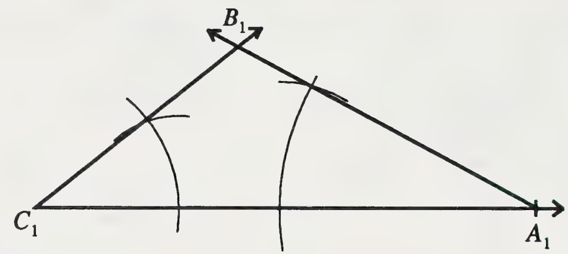




Activity 4

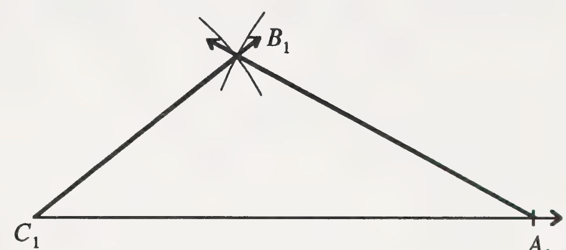
Construct a triangle congruent to a given triangle.

1.



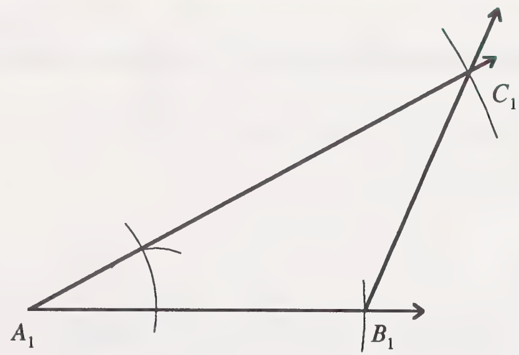
Side A_1C_1 was constructed first. Then angle A_1 and angle C_1 were constructed.

2.



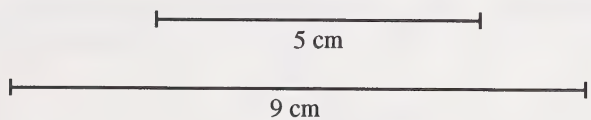
Side A_1C_1 was constructed first. Then sides A_1B_1 and C_1B_1 were constructed.

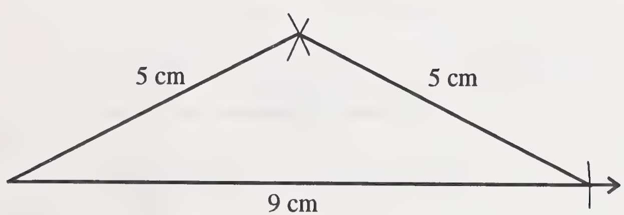
3.



Side A_1B_1 was constructed first. Then $\angle A_1$ was constructed. After $\overline{A_1C_1}$ was constructed, a segment was drawn to connect C_1 and B_1.

4.





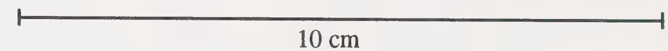
5. Since 1 km = 1000 m and 100 m is represented by 1 cm, you need to make a segment that is 10 cm long.

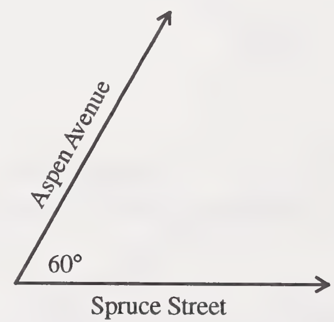
$$\frac{1\text{ cm}}{100\text{ m}} = \frac{x\text{ cm}}{1000\text{ m}}$$

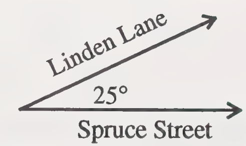
$$100x = 1000$$

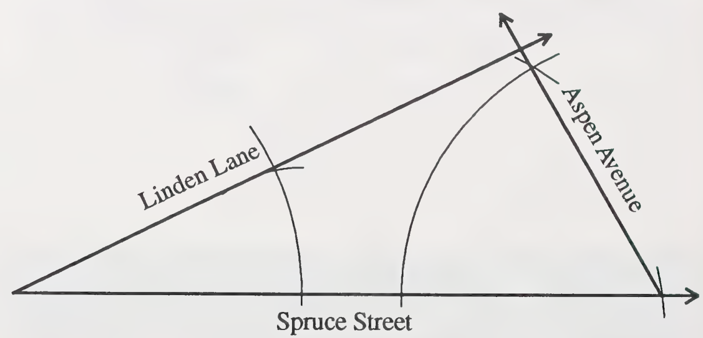
$$\frac{100x}{100} = \frac{1000}{100}$$

$$x = 10$$

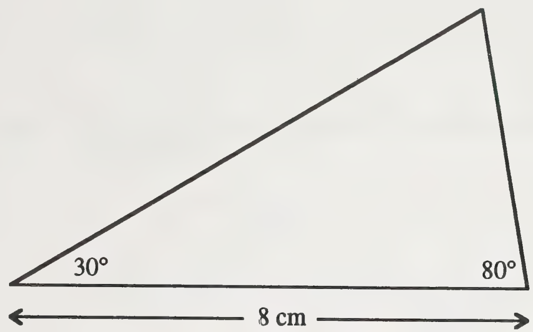


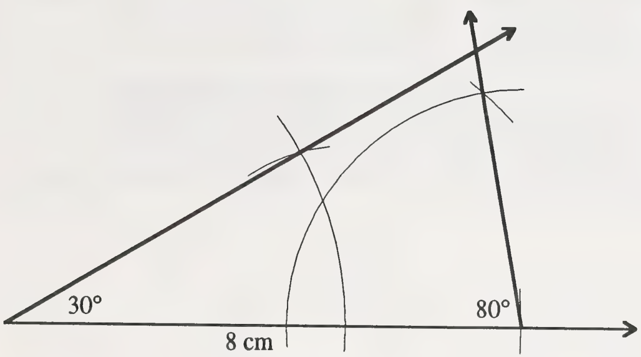






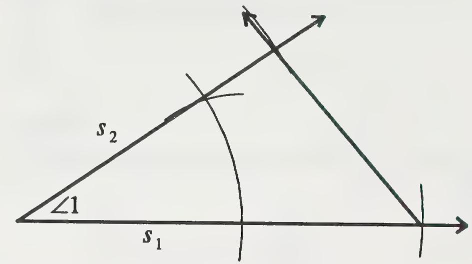
6. To draw a triangle using the given measures, you must use a ruler and a protractor.



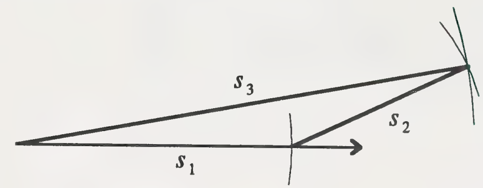


Extra Help

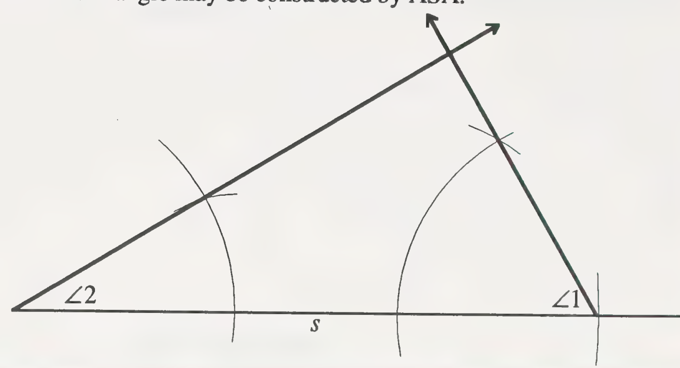
1. The triangle may be constructed by SAS.



2. The triangle may be constructed by SSS.

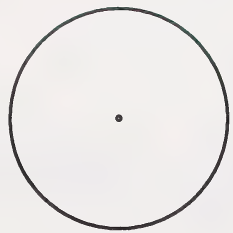


3. The triangle may be constructed by ASA.



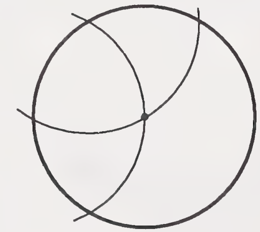
Extensions

1. Step 1: Draw a circle.

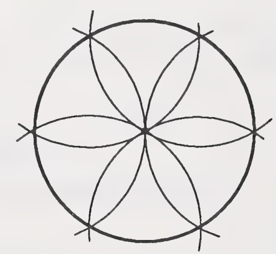


Step 2: Leaving the compass setting the same as in Step 1, place the point of the compass on the circle and make an arc intersecting the circle in two places. Notice that the arc passes through the centre of the circle.

Step 3: Leaving the compass setting as in Step 2, place the compass point on one of the intersections made previously and make another arc intersecting the circle in two places again.



Step 4: Continue in this fashion until the figure shown here is made.

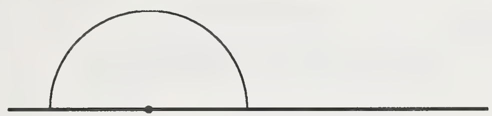


Step 5: Do some erasing and shading to get the finished diagram.

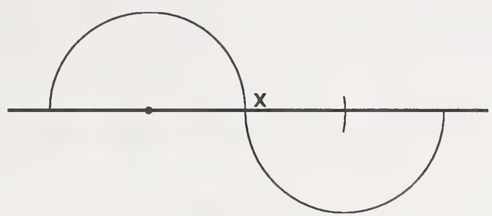
2. The following steps show how to make the figure.

Step 1: Draw a line segment.

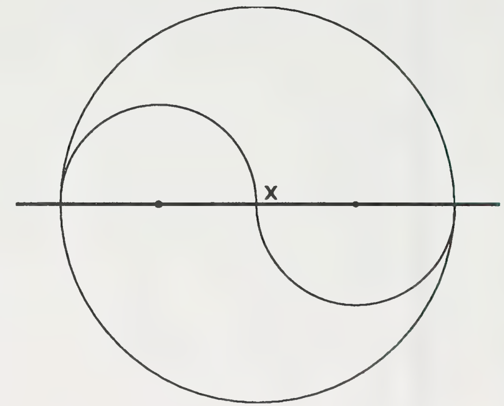
Step 2: At any point on the line segment, make a point and draw a semicircle.



Step 3: To draw the second semicircle, you must find its centre. To do this, keep your compass setting the same as in Step 2, place the compass point at **x**, and make an arc intersecting the line. The point of intersection will now be the centre for your second semicircle. Make this second semicircle below the line segment as shown.



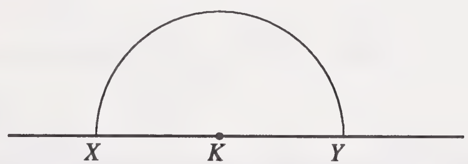
Step 4: Set your compass so that the pencil point is at one endpoint of one semicircle's diameter and the compass point is at the other endpoint of the same semicircle. Draw a circle using **x** as the centre.



Step 5: Do some erasing, shading, and colouring to vary the original design.

3. The method by which you make the figure is shown by the series of illustrations that follow.

Step 1: Draw a line segment. Using any point on the segment as your centre, draw a semicircle. Label your diagram as shown.



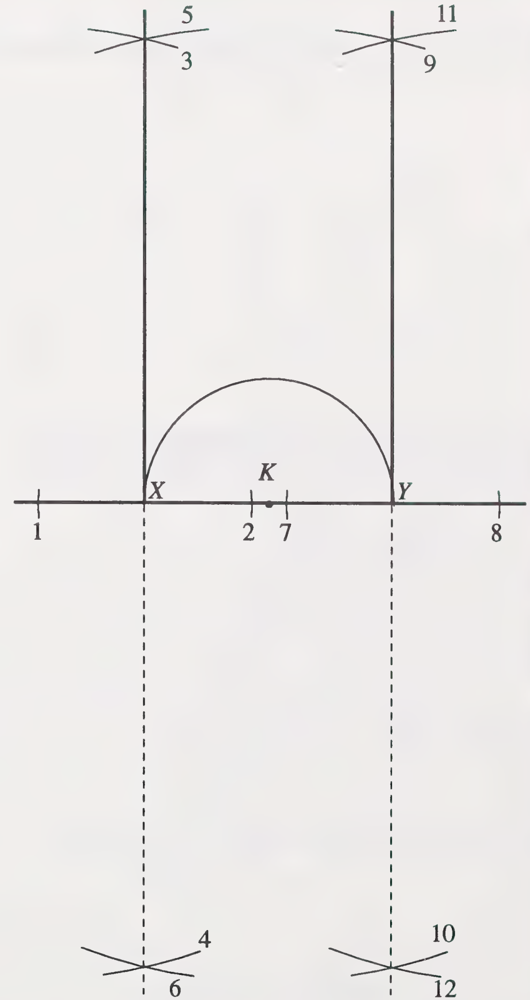
Step 2: Extend line segment XY if needed.

Step 3: Place the compass point on X and mark off arc 1 to the left of X and arc 2 to the right of X.

Step 4: Using point 1, make arcs 3 and 4. Using point 2, make arcs 5 and 6.

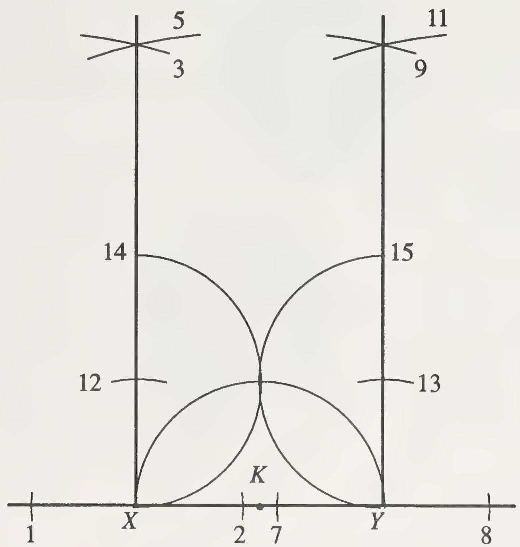
Step 5: Connect the points of the arcs' intersection to X. You will not need the dotted line later.

Step 6: Place the compass point on Y and repeat Steps 3, 4, and 5.



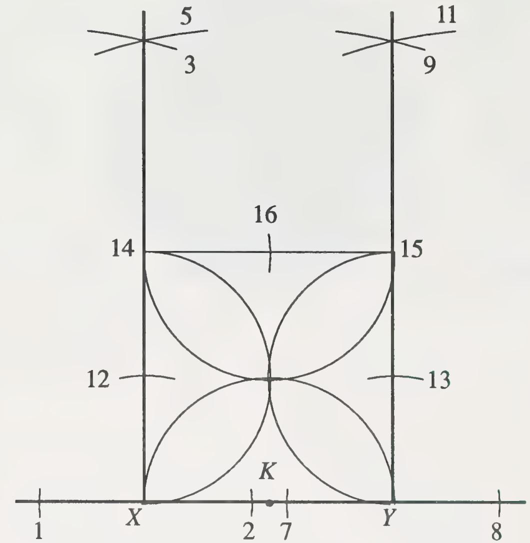
Step 7: For the next setting, put your compass point at X and the pencil point at K. Using this setting, put your compass point at X and draw arc 12. Use the same setting to draw arc 13 from point Y.

Step 8: Without changing the setting from Step 7, draw the left semicircle using point 12 as the centre. Draw the right semicircle using the same setting and point 13 as the centre.



Step 9: Join points 14 and 15. For the next setting, put your compass point at X and the pencil point at K. Using this setting, place the compass point at point 14 and make arc 16.

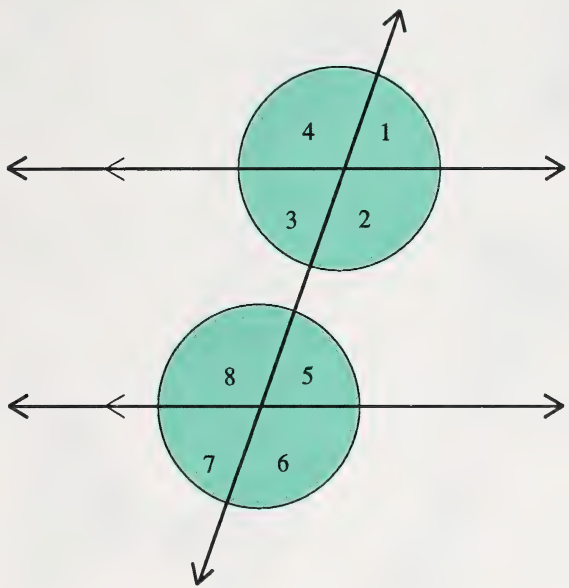
Step 10: Using the same compass setting as in Step 9, make the top semicircle using point 16 as the centre.

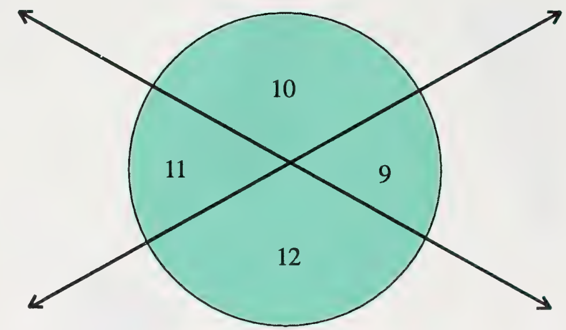


Step 11: After some erasing, shading, and/or colouring, you should end up with some interesting designs.

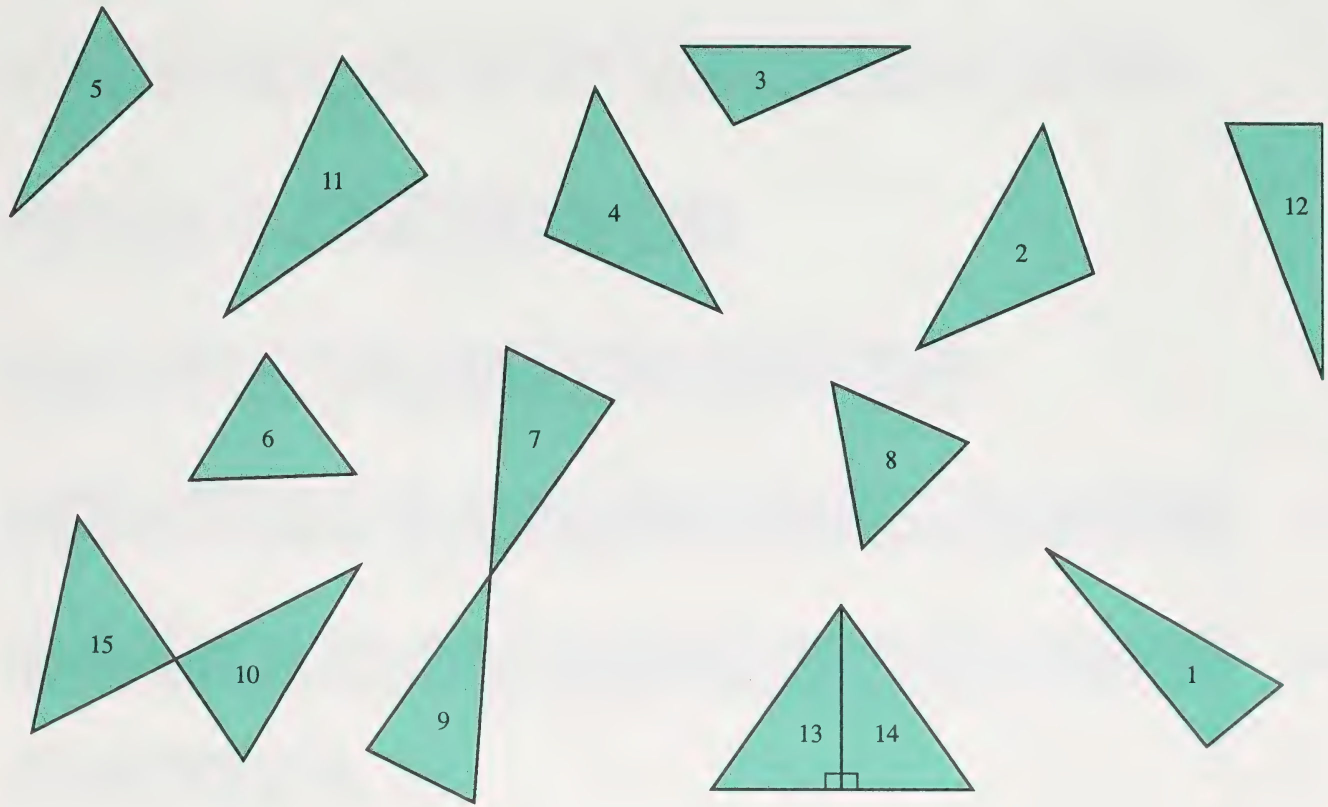
Appendix B
Manipulatives

Angles for Topic 1, Extra Help



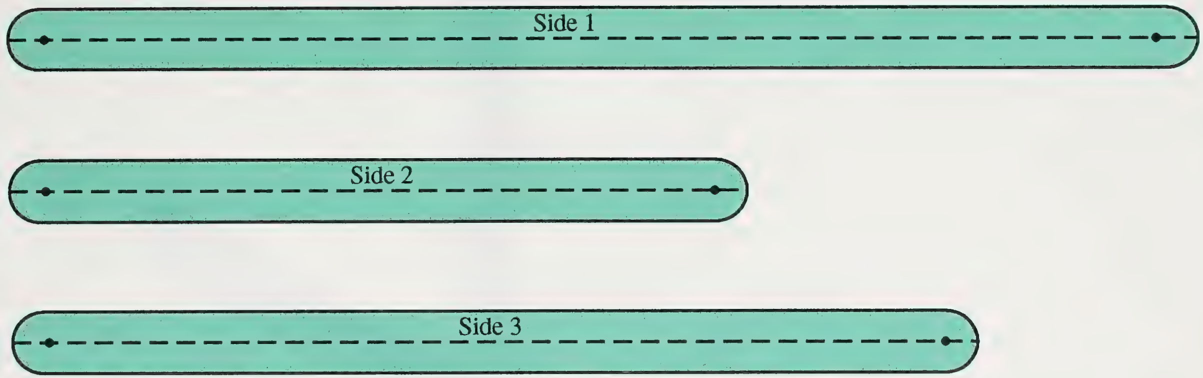


Triangles for Topic 3, Activity 1

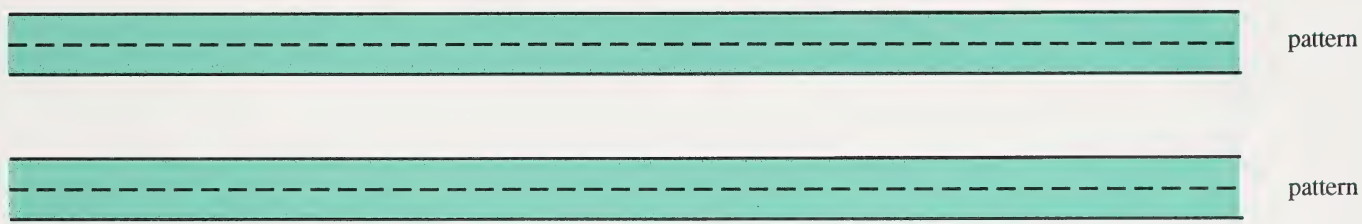


Building Figures for Topic 3, Activity 2

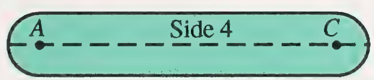
1.



2.

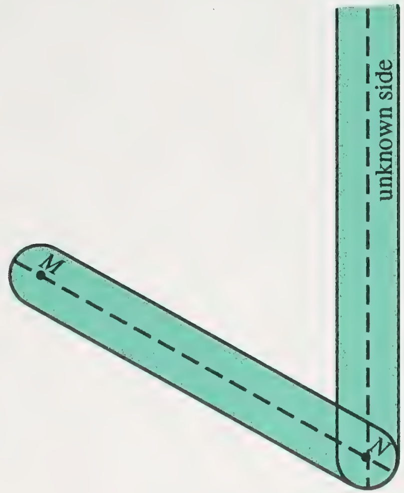


3.



Building Angle for Topic 3, Activity 2

4.



L.R.D.C.
Producer

Mathematics 13

9MA13P33

THIRD EDITION
1992